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AND PROF. J. WERTHEIMER, B.SC., B.A., F.I.C., F.C.S., *Principal of the Merchant Venturers' Technical College, Bristol.*

PRACTICAL MECHANICS

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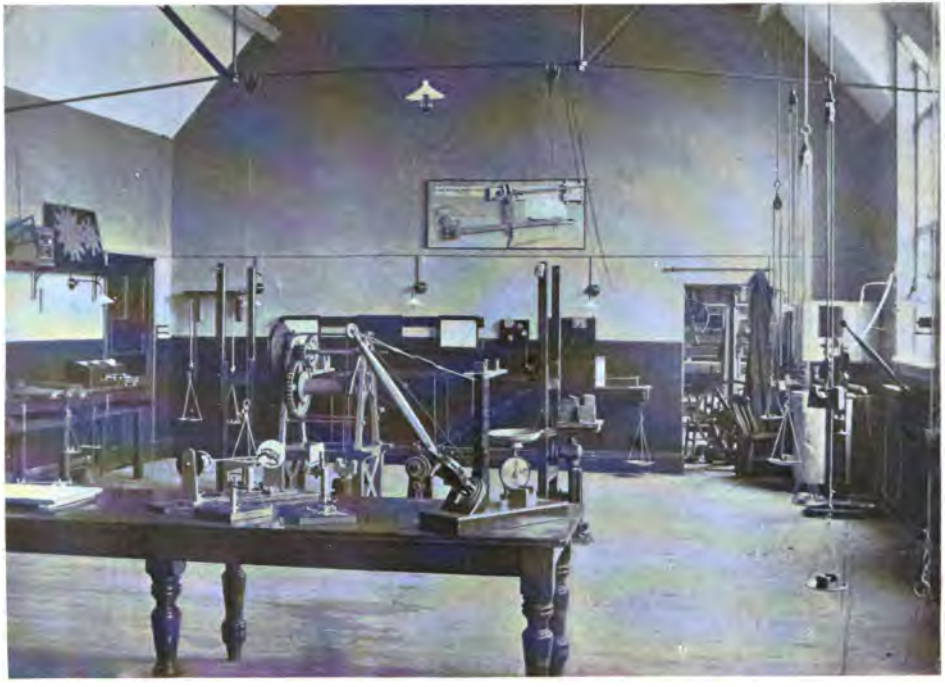

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PRACTICAL MECHANICS

AN ELEMENTARY MANUAL
FOR THE USE OF STUDENTS IN SCIENCE AND
TECHNICAL SCHOOLS AND CLASSES

BY

SIDNEY H. WELLS, WH. Sc.
A.M. INST. C.E.; A.M. INST. MECH. E.
PRINCIPAL OF THE BATTERSEA POLYTECHNIC, LONDON

WITH 75 ILLUSTRATIONS AND DIAGRAMS
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MACHINES AND APPARATUS AND NUMEROUS GRAPHICAL
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PREFACE

THIS book is intended as a Laboratory hand-book for the use of students in Science and Technical Schools and Classes, and as a guide to teachers in arranging machines and apparatus, and courses of Laboratory work in Practical Mechanics.

For the past ten years I have given considerable attention to methods of teaching elementary mechanics in a practical way, and with such varied students as are to be found in a great public school, a university college, and a technical institute. My experience has shown that with large classes it is almost impossible to have all the students working at the same experiment, or even at the same kind of experiment, at the same time; and I have felt the need of some assistance to the teacher which should start the student on his work, describe the apparatus to be used, give him sufficient directions to carry out the experiment in the right spirit of inquiry, suggest to him how to analyse the result and what he should have learned from it, and then to provide a useful test of his work in the form of suitable exercises illustrating the experiment. With school classes of moderate size, where the theory and practical lessons run concurrently, the difficulty referred to is scarcely felt, except as

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the work becomes advanced and the apparatus limited; but with older students, and especially with evening students where the conditions named are scarcely possible, where the classes are necessarily large and the number of teachers small, it has seemed to me impossible to do good work without such assistance as I have referred to. Then, again, teachers are desirous of conducting experimental classes in Mechanics, and are needing information as to the cost, construction, and use of the necessary machines and apparatus, and the results obtained by them. Add to this the facts that Laboratories for practical mechanics are rapidly increasing in number, and that students preparing for London Matriculation and for the Theoretical and Applied Mechanics examinations of the Science and Art Department are expected to *have carried out experiments for themselves*, and there exists, I think, more than sufficient reason for "yet another text-book."

At the outset I was met by the difficulty of choosing between the methods of "verification" and "discovery"—whether it was better to state a law and require the student to verify it by experiment, or to ask him to perform an experiment and deduce the law from it. Neither is universally applicable, but as a believer in the method of "learning by doing," whenever practicable I have unhesitatingly adopted the latter method. To be strictly true to this principle, the book should not, of course, tell the student what he should have learned from the experiment, and, indeed, my great difficulty all through has been how to avoid *telling the student too much.* Yet it would have been impracticable to omit any records of

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results, or any discussions of the conclusions to be drawn from them, besides which I hope that the "records" and "discussions" may not be without real educative value.

For the benefit chiefly of teachers, I have included somewhat detailed descriptions of the machines and apparatus, together with reproductions from photographs, for which I am much indebted to the kindly assistance of Mr. Joseph Harwood, Secretary of the Polytechnic, and have added appendices showing the cost and other details, together with a suggested course of work for elementary students in Applied Mechanics. I am no believer in costly and elaborate apparatus, or in frictionless pulleys and toy models, preferring rather the actual machines as used in practical work, and such apparatus as can be easily adapted from commonly obtained things. My experience has shown that, with the kind of apparatus illustrated in this book, it is possible to obtain results quite within the ordinary experimental range of accuracy, in proof of which the results herein recorded will be evidence, although an exception must be made for the results in the chapter on "Beams."

The book contains all the Mechanics part of the Elementary Science syllabus of the Headmasters' Association, and all that is necessary for the London Matriculation examination and for the Elementary course in Applied Mechanics of the Science and Art Department. In many subjects it covers the Advanced Stage also. The exercises are selected from past papers set in these and other examinations, and it is hoped they may be found useful. The records of experiments are chiefly taken from students' note-books.

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My thanks and acknowledgments are due to my students—Messrs. C. B. Cunnington, A. P. Morris, and F. J. Sharr—for working through the experiments, and to my late Assistant—Mr. W. F. Nixon, A.R.C.S.—for assistance in this and other work and for reading through the proofs, and especially to Mr. W. W. F. Pullen, Wh. Sc., Assoc. M. Inst. C.E., M. Inst. Mech. E., Head of the Engineering Department at the South-West London Polytechnic, for reading through the manuscript and proof sheets, and for many valuable suggestions.

SIDNEY H. WELLS.

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PRACTICAL MECHANICS

CHAPTER I.

GRAPHIC STATICS. PARALLELOGRAM AND TRIANGLE OF FORCES.

STATICS is that part of mechanics which deals with the action of forces on bodies under such conditions that the bodies remain at rest.

The following problems are concerned with forces acting in the same plane, or *Complanar Forces.*

Graphic Statics. Statical problems may be solved by means of geometrical figures or diagrams in which the forces are represented by lines. This is known as the "graphic method," or as "graphic statics," and the problem is said to be worked graphically.

Units of Force. The units of force adopted are the "weight of one pound" (1 lb. weight), or the "weight of one gramme" (1 gm. weight). It is better to use the word *weight* in order to distinguish between the "mass of one pound" and "the weight of one pound," as it is only the latter which is a unit of force. The distinction is not so necessary in applied mechanics as in theoretical mechanics; hence it is more usual to use the expression "a force of one pound," rather than "a force of one pound weight."

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Specification of a Force. A force is usually regarded as being completely specified when we know :—

(1) Its point of application, or the point at which it acts.
(2) The direction in which it acts, or its line of action.
(3) The magnitude of the force as 7 lbs. weight or 100 gms. weight.

Some confusion may exist as to what is really meant by the *direction* of a force. Suppose a force acts in a vertical direction, it may act either upwards or downwards, and it is necessary to know which. Does the word *direction* mean its position, *i.e.* that it acts vertically, or does it mean that it acts upwards or downwards? It is more usual to mean the former; and for this reason many writers use the word "*sense*" to indicate whether the action is upwards or downwards. Hence it is sometimes stated that in addition to the above conditions it is necessary to know the "*sense*" of the force.

Graphic Representation of a Force. A force is completely represented by a straight line when :—

(1) One end of the line coincides with the point of application of the force.

(This is not really necessary, as the student will see later; nor should it make him forget that a force may act anywhere along a line without change in effect.)

(2) The direction of the line represents the direction or line of action of the force.
(3) The length of the line represents the magnitude of the force to some scale, as 1″ to 10 lbs. weight, or 1 cm. to 5 gms. weight, etc.

To these conditions a fourth is sometimes added; viz, that an arrow upon the line shall show its *direction* or *sense* along the line. This is a very common and convenient way of representing how the force acts along the line; but it is not really necessary. For if we agree to call a line by two letters, as AB or CD, etc., and the line represents a force, then we

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may agree that if we speak of it as AB, we shall mean that the force acts along it from A to B; and if we speak of it as BA, that it acts along it from B to A.

Thus the line AO, Fig. 1, represents a force of 14 lbs. weight acting at the point O in a direction making an angle of 45° with the horizontal. Scale, 1″ to 10 lbs. weight. We may mark an arrow upon it to show that it acts from A to O, although this is really not necessary if we speak of it as AB.

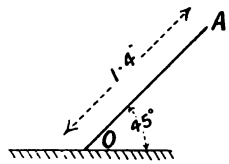

FIG. 1.

NOTE.—In drawing lines to scale, it is better to use a scale of tenths, so that decimal parts may be easily and accurately represented. The scale used should always be stated.

EXAMPLES.

Represent the following forces graphically :—

1. A force of 76 gms. acting at a point O in a direction making an angle of 60° to the horizontal. (Scale, 1 cm. to 10 gms.)

2. Four forces of 22, 15, 18·5, and 25 lbs. weight respectively acting at a point O. The direction of the first force is horizontal; the angle between the first and second is 60°; between the second and third 75°; and between the third and fourth 75°. (Scale, 1″ to 10 lbs.)

3. Two forces of 21 and 14·5 lbs. acting at a point O, so that the tangent of the angle between their lines of action is $\sqrt{3}$.

DEFINITIONS. The following definitions should be understood and remembered :—

Resultant. *The resultant of two or more forces is that single force which, if acting alone, would produce the same result as the two or more forces acting together.*

Equilibrant. *The equilibrant of two or more forces is that single force which, acting together with the two or more forces, produces equilibrium, or* **rest.** *The effect of the equilibrant is therefore exactly equal and opposite to the combined effect of the two or more forces.*

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The equilibrant of any number of forces is equal and opposite to the resultant of these forces. For this reason the equilibrant is sometimes called the **anti-resultant.**

Component. *A single force can be resolved into two forces whose combined effect is the same as that of the single force. These two forces are called components of the single force.*

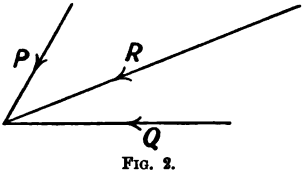

FIG. 2.

If R is the resultant of the two forces P and Q, then these forces are said to be components of R.

When the components are at right angles to each other, they are termed **rectangular components.**

Resultant of Forces acting at a Point. *The simplest case of two forces acting at a point is when they act along the same straight line. If they act in the same direction along the line, their Resultant is equal to their* **sum**; *and if they act in opposite directions along the line, their Resultant is equal to their* **difference.**

EXPERIMENT I. **To find the Resultant of two forces of $1\frac{1}{2}$ and 2 lbs. weight acting at a point along lines inclined at an angle of 90°.**

APPARATUS (Fig. 3). A board about $12'' \times 10''$ [1] is fixed with its plane and long edges vertical, and light, well-balanced, easy running pulleys about $2''$ to $3''$ in diameter are fixed at each top corner. The rims of the pulleys should be provided with a deep narrow groove. Three soft flexible cords (preferably silk about $\frac{1}{16}''$ diameter) have one end of each loosely tied to a small brass ring (or they may be simply knotted together), the other ends being free for attachment to weights. The cords are placed as shown in Fig. 3, and weights are tied to the free ends to represent forces, or they may be placed in small light scale-pans.

[1] For later experiments with the pulley-boards it is better to have them about $31'' \times 23''$ (Imperial size).

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NOTE.—The pulleys should be so fixed that the cords are not more than $\frac{1}{16}$" clear of the board Care should be taken to see that the pulleys turn quite freely, and the board should be strongly tapped, or the centre cord sharply pulled once or twice, in order to overcome any sticking of the pulleys, and to allow the cords to assume their correct position.

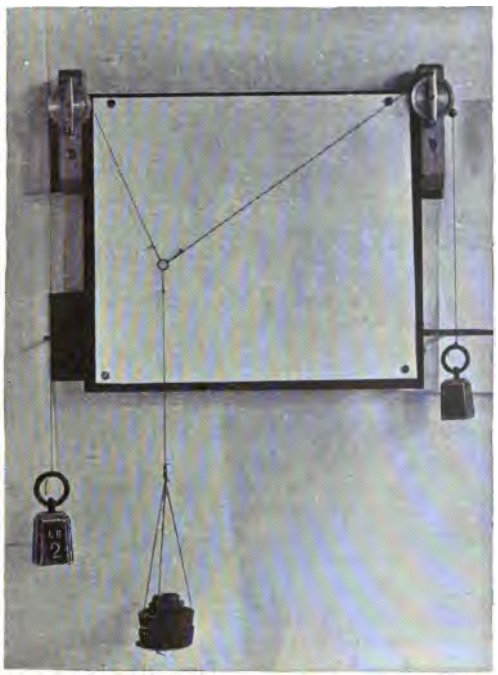

FIG. 3.

Method. (1) Take a piece of ordinary drawing paper or unruled foolscap, and draw upon it two lines OA and OB meeting at a point O at an angle of 90° (see Fig. 4). (2) Tie weights of 2 lbs. and $1\frac{1}{2}$ lbs. to the left and right hand cords respectively, and tie a scale-pan to the middle cord. (3) Place the paper behind the cords, and add weights to the scale-pan until the cords coincide with the lines upon the paper, so that the angle between the cords is now 90°. (4) Pin the paper to the board, and mark a point C immediately under the middle cord, so that a line drawn from O to C coincides with the cord. (5) Remove the

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paper, weigh the scale-pan, and write on the paper the magnitude of the forces on each cord, and show by an arrow on the lines OA, OB, OC the direction of the forces. Join OC. The paper will now be as shown by the dark lines in Fig. 4. (6) Along OA mark off a distance OD to represent 2 lbs. weight to any convenient scale, say 1″ to 1 lb. weight, and along OB mark off a distance OE to represent $1\frac{1}{2}$ lbs. weight. Produce CO upwards, and draw lines DF and EF parallel to OE and OD respectively. Then the figure $DOEF$ is a parallelogram, and OF is its diagonal. (7) Measure the diagonal OF to the same scale as used for OD and OE. *It will be found to represent the force in the cord OC, that is $2\frac{1}{2}$ lbs. weight.* Now produce the line CO upwards, and it should be found to coincide with the line OF.

What the Experiment teaches. The weights on the middle cord keep the ring in equilibrium; therefore the force of $2\frac{1}{2}$ lbs. weight acting downwards is the equilibrant of the two forces of 2 and $1\frac{1}{2}$ lbs. acting upwards at an angle of 90°. But the resultant of a number of forces is equal and opposite to their equilibrant. Therefore the line OF represents a force which, if it acts upwards, is the resultant of the two forces of 2 and $1\frac{1}{2}$ lbs. represented by the lines OD and OE; and this line is the diagonal of a parallelogram, of which these lines OD and OE are adjacent sides, and it passes through the point O, at which the lines meet.

Students who possess a knowledge of theoretical mechanics will recognize in this result the principle known as the parallelogram of forces; but they must not make the mistake of regarding the above experiment as a *proof* of this principle. No single experiment can be said to prove a general proposition, although it may well be regarded as a verification of it. The experiment has been given in the above form in order to lead up to and make clear the following statement of the proposition of the parallelogram of forces, the proof of which is a part of theoretical mechanics.

Parallelogram of Forces. *If two forces acting at a point be represented in magnitude and direction by the adjacent sides of a parallelogram drawn from the point, then their resultant is represented in magnitude and direction by the diagonal of the parallelogram passing through the point.*

Record of Experiment. The experiment just described should be entered in the Laboratory note-book somewhat as follows :—

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Experiment to find the resultant of two forces of $1\frac{1}{2}$ *and* 2 *lbs. weight acting along lines inclined at an angle of* 90°.

The Resultant by experiment is $2\frac{1}{2}$ lbs. weight. $O\,D$ represents 2 lbs. weight, $O\,E$ represents $1\frac{1}{2}$ lbs. weight. By completing the parallelogram $D\,O\,E\,F$, the diagonal $O\,F$ represents $2\frac{1}{2}$ lbs. weight. Therefore the Resultant is equal to the diagonal of the parallelogram of which the two forces are adjacent sides.

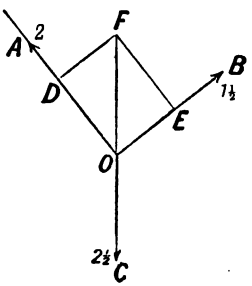

FIG. 4.

EXPERIMENT II. **Verify by experiment the parallelogram of forces, writing down a description of the experiment and its result. Use weights of 2 and $1\frac{1}{2}$ lbs. and of $2\frac{3}{4}$ lbs. for the centre cord.**

EXPERIMENT III. **To find one of the two components of a force when the force and one component are given. A force of 3 lbs. acting vertically is resolved into two components, one of which is $2\frac{1}{2}$ lbs., the angle between them being 100°. Find the other component by experiment, and verify graphically.**

EXERCISES.

The following exercises are to be worked graphically :—

1. State how to find the resultant of any number of forces acting along a line.

Draw a straight line $A\,B$, and let three forces of 15, 10, and 33 units respectively act along it ; the former two act from A to B, the last acts from B to A. Find their resultant.

2. Two equal forces act at an angle of 75°, and their resultant is 20 lbs. Find the forces.

3. Resolve a force of magnitude 12, acting horizontally from left to right, into two components, one of which is a force of magnitude 25, acting vertically upwards.

4. Show that the resultant of two forces acting at a point is nearer to the greater force.

5. Show that as the angle between two forces is increased their resultant is diminished.

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6. Two forces of 5 and 11 act at angles of 60°, 90°, 120° respectively. Compare their resultants in the three cases.

7. Draw lines AB, AC, such that BAC is an angle of 70°; a force of 12 units acts from A to B, and a force of 15 units from A to C Find by construction their resultant; and if the resultant acts from A to D, find from your diagram the number of degrees in the angle BAD.

Specify the resultant. If the question had been, Find the force which will balance the two given forces, what difference would it make in the specification?

8. ABC is a triangle whose sides BC, CA, AB are 7, 4, 5 units long. At A two forces act, one of 8 units from A to C, and one of 10 units from A to B. Draw a straight line through A, to represent their resultant in all respects, and state the number of units of force in the resultant.

9 Draw two lines AB and AC containing an angle of 120°, and suppose a force of 7 units to act from A to B, and a force of 10 units from A to C. Find by construction the resultant of the forces, and the number of degrees in the angle its direction makes with AB.

10. Draw two lines Ox, Oy at right angles to each other. Two forces act at O, one of 7 units from x to O, and one of 10 units from O to y. Draw to any scale straight lines to represent these forces; draw to the same scale the straight line (OR) that represents their resultant, and find from the diagram the number of units of force in the resultant, and the number of degrees in the angle xOR.

11 Draw an angle AOB of 120°, and draw OC within the angle AOB, so that AOC may be 45°. If a force of 100 units acts from O to C, find its components along OA and OB.

12. Draw an equilateral triangle ABC. A force of 10 units acts from A to B, and one of 15 units from A to C. Find their resultant. Also find what their resultant would be if the force of 15 units acted from C to A.

13. Draw two straight lines OA, OB, containing a right angle at O. Within the right angle draw OP, such that AOP is an angle of 35°. A force of 18 units acts from O to P. Find its rectangular components along OA and OB.

14. Draw a diagram showing the resultant of two forces equal to the weight of 7 and 11 lbs., acting on a particle, with an angle of 60° between them; and by measuring the resultant, find its numerical value. Find two forces, at right angles to each other, which could maintain equilibrium with the above.

15. A boat is being towed by a rope making an angle of 30° with the boat's length. The resultant pressure of the water on the boat and the tension in the rope is equal to the weight of half a ton. Find the resultant force in the direction of the boat's length.

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EXPERIMENT IV. **To find by experiment the conditions of equilibrium of three forces of $3\frac{1}{4}$, $2\frac{1}{2}$, and 2 lbs. weight acting at a point.**

APPARATUS. The same as used for Experiment I.

Method. (1) Tie weights of $2\frac{1}{2}$ and 2 lbs. to the end cords, and $3\frac{1}{4}$ lbs. to the middle cord, and place a sheet of paper behind them marking their positions when in equilibrium exactly as in Experiment I. (2) Remove the paper, and draw lines as before, and letter as shown, so that the results appear as in Fig. 5*a*. (3) Anywhere on the paper

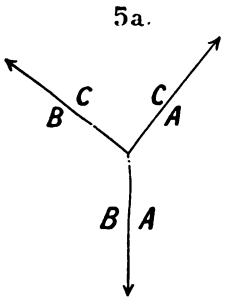

FIG. 5*a*.

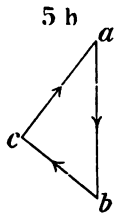

FIG. 5*b*.

FIG. 6.

removed from Fig. 5*a* draw a line *a b* (Fig. 5*b*) parallel to the cord *A B*, and of a length to represent the weight on this cord of $3\frac{1}{4}$ lbs. to any convenient scale. From one end as *b* draw a line parallel to *B C*, and from the other end *a* draw a line parallel to *C A* meeting the first line in the point *c*. (4) Measure *b c* and *c a* to the scale used in drawing *a b*. They will be found to represent the forces in the cords *B C* and *C A* respectively. (5) Mark arrows upon the triangle *a b c* showing the direction of the forces, and notice that they all go round the same way, that is, for Fig. 5*b* in a clockwise direction.

RECORD OF EXPERIMENT. *To find by experiment the conditions of equilibrium of three forces of* $3\frac{1}{4}$, $2\frac{1}{2}$, *and* 2 *lbs. weight acting at a point.*

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Figs. 5*a*; *b*.[1] ab was drawn parallel to AB, and to represent $3\frac{1}{4}$ lbs. weight. bc was drawn parallel to BC, and ca to CA; then length of $bc = 2''$, and represents 2 lbs. weight; and of $ca = 2\frac{1}{2}''$, and represents $2\frac{1}{2}$ lbs weight.

NOTE.—If the experiment is given in the form of finding the conditions of equilibrium of any three forces, the following apparatus may be used:

A. (Fig. 6.) A board $31'' \times 23''$ is fixed with its plane vertical. Two strong hooks are screwed into the top corners, and from these hang light spring balances reading to 8 lbs by 2 ozs.[2] A cord is tied to the hook of

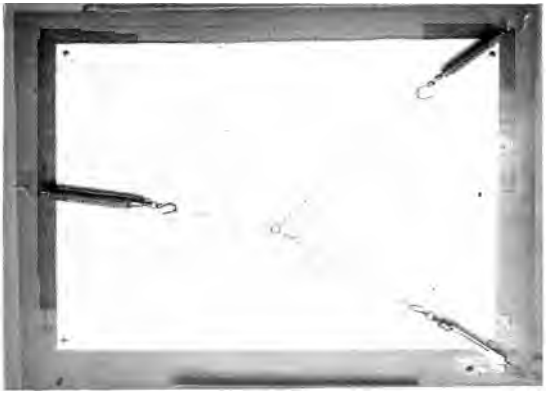

FIG. 7.

each balance, and the other end is tied to a small ring as before. A 4 lb. weight is attached to the third cord as shown.

B. (Fig. 7.) A board the same size as before with three spring balances as shown. With this arrangement the board may be laid upon the table.

What the Experiment teaches. From the working of the experiment it is obvious that the three forces are in equilibrium. The triangle abc was drawn with one of its sides ab parallel to the cord AB, and of a length to represent the

[1] NOTE.—The structure and force diagrams, as in Figs. 5*a*, *b*, should always be entered in the student's note-book as a part of the record of the experiments.

[2] For particulars of spring balances see Appendix I.

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force in this cord; and the other two sides bc and ca were drawn parallel to the cords BC and CA respectively. We then found that the length of these sides bc and ca represented the forces in the cords to whose directions they were parallel. From this we may conclude that when three forces acting at a point are in equilibrium, a triangle can be drawn having its sides parallel to the direction of the forces, and that the length of the sides will be proportional to the magnitude of the forces to which they are parallel. Also that the arrows showing the direction of the forces all go round the same way, that is, in the same direction as the hands of a clock.

Students should now be able to understand the following statement of the general proposition of the equilibrium of three forces acting at a point, of which the above experiment may be regarded as a verification.

Triangle of Forces. *If three forces acting at a point can be represented in magnitude and direction by the sides of a triangle taken in order, they are in equilibrium;* and conversely,

If three forces acting at a point are in equilibrium, and a triangle be drawn having its sides parallel to the direction of the forces, the sides will be proportional to the forces to whose directions they are parallel.

Structure Diagram and Force Diagram. In working the last experiment it was found convenient to draw two separate diagrams, Fig. 5*a* and *b*. Fig. 5*a* shows the position of the point at which the forces act, and of the lines along which they act; but it does not represent the magnitude of the forces, nor does it show, except by the arrows, their direction or sense along the lines. Fig. 5*b* shows only the magnitude and sense direction of the forces. The first of these is sometimes called a "space diagram," and sometimes a "frame diagram," but we shall call it a "*structure diagram*," because it represents the structure in which the forces are acting. The second we shall call by its usual name of "*force diagram.*"

Method of Lettering Diagram. By referring to the structure diagram of Fig. 5*a*, the student will see that a letter has been placed in each space between the lines of action of

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two forces; thus the space between the line of action of the middle force and that of the right-hand force is marked *A;* the space between the line of action of the middle force and the left-hand force is marked *B;* and the space between the lines of action of the right- and left-hand forces is marked *C.* We can then speak of the pull in the middle cord as the force *A B;* and of the pull in the other cords as the force *B C* and *C A.* Also we can speak of the lines of action of these forces as *A B*, *B C*, and *C A* respectively. Notice that in doing this we have gone round the diagram in the same direction; that is, in a "clockwise" direction.

Now in drawing the force diagram in Fig. 5*b* we call the line which represents the middle force "the line *a b*," and we mark the upper end of the line *a*, and the lower end *b*, because the force acts downward. In the same way the line in the force diagram which is parallel to the cord *B C* is marked *b c*, and the line which is parallel to the cord *C A* is marked *c a*. That is, we use the same letters for the force diagram as for the structure diagram; the difference being that we use *capital* letters for the latter, and *small type* letters for the former. Now notice that we speak of the force *a b*, and not *b a*, because the force acts from *a* to *b;* and as we must go round the diagram in the same way, that is, in a clockwise direction, we speak of the force *b c*, meaning that it acts "from *b* to *c;*" and of the force *c a*, meaning that it acts "from *c* to *a*." From the structure diagram it is obvious that the forces do act in this way, but in more complicated structures it is not always possible to see how the forces act, except by some such method as we have described. This system of lettering is known as "Bow's notation," and should be adopted by the student in all succeeding problems in graphic statics. Notice that it enables us to tell which way the force is acting without drawing arrows upon the force diagram; for example, suppose we wish to know the direction of the force in the left-hand cord, we look on the structure diagram, and see that the cord is *B C*, not *C B*, because we take the letters in a "clockwise" order, and the letter *B* comes first. We then look on the force diagram for the line *b c*, and knowing that *b c* means "from *b* to *c*," we see that the force acts from *b* to *c;* that is, the cord is pulling the ring upwards to the left hand. Similarly the right-hand cord is *C A*, which means that in the force diagram the force

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acts from c to a, or upwards to the right. It is sometimes convenient to place a letter on each side of the line of action of a force, thus using each letter twice, as in Fig. 5*a*; but it is generally sufficient to simply write the letter once only in the space between the lines of action of the forces.

Another proof of the "Triangle of Forces." The proposition of the "triangle of forces" can be obtained directly from the "parallelogram of forces" as follows:—

Draw lines $O\,A$, $O\,B$ at any angle, say of 60°, to represent the line of action of two forces P and Q acting at the point O. Let P be 2 lbs. weight, and Q be 3 lbs. weight, and make the length of $A\,O$ to represent P, and of $B\,O$ to represent Q to any scale, say 1″ to 1 lb. weight. Complete the parallelogram $A\,O\,B\,C$,

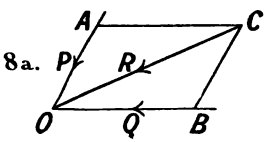

FIG. 8*a*.

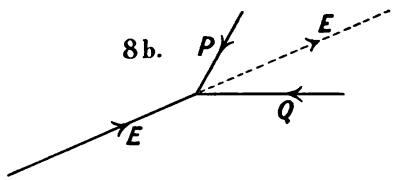

FIG. 8*b*.

and draw the diagonal $C\,O$. Then by the "parallelogram of forces" the length of $C\,O$ represents the magnitude of the resultant of the forces P and Q. If the forces P and Q act towards the point O, then the resultant R must also act towards the point O, as shown by the arrows. But the "equilibrant is equal and opposite to the resultant;" therefore if the force which is represented by the length of the diagonal $O\,C$ acts from O to C, instead of from C to O, it will be the equilibrant of the two forces P and Q, and the three forces P, Q, and R will be in equilibrium if P and Q act from A and B towards the point O, and if R acts from O to C. Draw another diagram, as in Fig. 8*b*, to represent these forces when acting in this way; and notice that the equilibrant E may act either as shown by the full line or by the dotted line. Notice also that the line $B\,C$ is equal and parallel to the line $A\,O$, and therefore represents the force P. Therefore the triangle $O\,B\,C$ has each

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of its sides respectively equal and parallel to the three forces P, Q, and E, which, when acting as in Fig. 8*b*, are in equilibrium. The arrows which represent the direction of the forces P, Q, and E all go round the same way when drawn upon the triangle $O\,B\,C$; that is, from B to O, from O to C, and from C to B. The student should see from this that in finding the resultant of two forces it is unnecessary to draw the whole parallelogram, as one-half of it is sufficient.

EXERCISES.

The following exercises are to be worked graphically :—

16. Three equal forces act in one plane on a point, in such a way that each of them makes an angle of 120° with each of the other two. Prove that the forces will balance.

17. Show by a diagram drawn to scale the lines along which three forces of 13, 12, and 5 units must act if they are in equilibrium, and find from the diagram the angle between each pair of forces.

18. Show that three forces are in equilibrium which are represented in magnitude and direction by the straight lines drawn from the angular points of a triangle to the middle points of the opposite sides.

19. Draw an equilateral triangle $A\,B\,C$, and produce $B\,C$ to D, making $C\,D$ equal to $B\,C$. Suppose that $B\,D$ is a rod (without weight) kept at rest by forces acting along the lines $A\,B$, $A\,C$, $A\,D$, given that the force acting at B is one of 10 units acting from A to B. Find by construction the other two forces, and specify them completely.

Mention the points that go to the specification of a force.

20. Enunciate the proposition known as the triangle of forces.

One end of a string is attached to a fixed point A, and after passing over a smooth peg B in the same horizontal plane, sustains a weight of P lbs.; a weight of 50 lbs. is now knotted to the string at C, midway between A and B. Find P, so that in the position of equilibrium $A\,C$ may make an angle of 60° with $A\,B$.

MISCELLANEOUS EXPERIMENTS ON THE TRIANGLE OF FORCES.

The following experiments and exercises include the principle of the triangle of forces, and present the usual condition of including three forces acting at a point, the direction of all three, and the magnitude of one only being known.

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Tension in Cords.

EXPERIMENT V. **A weight of 4 or 7 lbs. is tied to the lower end of a vertical cord. The weight is then pulled horizontally by a second cord until the first cord is inclined to the vertical at an angle of (*a*) 30°, (*b*) 45°. Find experimentally and graphically the pull in the cords.**

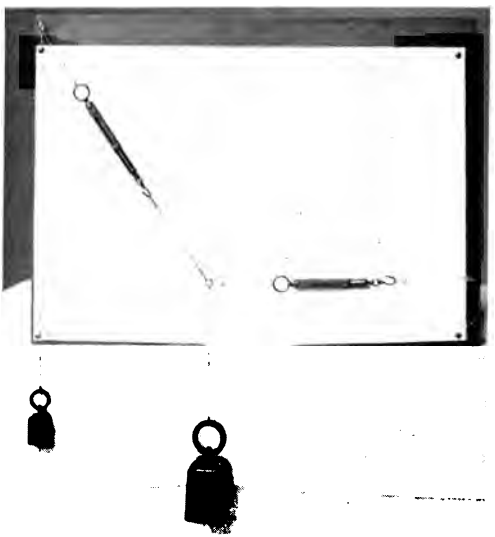

FIG. 9.

APPARATUS. (Fig. 9.) A board about 31″ × 23″ with hooks. Two spring balances reading to about 8 lbs. by 2 ozs. A weight, cord, and drawing paper.

Method. (1) Tie the weight to one end of cord, and tie the other end to the hook of the upper balance, as shown in Fig. 9. (2) Tie a second cord to a point in the vertical cord a few inches above the weight, and fasten the other end to the hook of the lower spring balance, and tie the ring of the balance by a separate cord to the hook as shown, so that the cord may be shortened or lengthened as desired. (3) Draw two lines upon a sheet of drawing paper meeting at an angle of 30°, and hang a small

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weight or "plumb-bob" by a piece of cotton or thin string from the hook of the upper balance. (4) Place the paper behind the vertical cord, and shorten the horizontal cord until the vertical cord and the "plumb-bob line" coincide with the lines drawn upon the paper; that is, until the angle between them is 30°. See that the other end is horizontal by measuring the height of its ends from the edge. It can be adjusted by altering the length of the inclined end. (5) Take the reading of the balances. (6) Draw a "structure diagram" of the arrangement, that is, a diagram of the cords and weight, to give the angles between them (see Fig. 10*a*). (7) Draw a "force diagram" to a force scale, say of 1″ to 1 lb. weight, as follows (see Fig. 10*b*):—draw a vertical line *a c* to represent the line of action of the weight, and make its length equal to the magnitude of the weight to the selected scale; from the end *c* draw a line parallel to the horizontal cord *BC*, and from the other end *a* draw a line parallel to the other end *CA*, to meet the first line in the point *b*. Then *a c b* is a triangle of forces, for each side is parallel to one of the forces; and as one side *a c* was made to represent one of the forces, therefore the other sides respectively represent the forces to whose line of action they are parallel; that is, the length of *c b* represents the pull in the string *CB*, and the length of *b a* represents the pull in the string *BA*. (8) Measure the sides *cb*, *ba* of the force triangle to the scale used, and write down the magnitude of the forces they represent, also write down the magnitude of the forces as read from the balances, so that the two results may be compared. (N.B. The reading of the balances must be taken before the cords are fixed in position, and allowance made if they show any pull; thus, if the reading of the horizontal balance in the cord *CB* is 3 ozs. before fixing, and 5 lbs. after fixing, then the pull due to the weight is 5 lbs. – 3 ozs., or 4 lbs. 13 ozs.) (9) Repeat the experiment for the angle of 45°.

Record of Experiment. Students should enter this experiment in their Laboratory note-books as follows:—

To find graphically and experimentally the pull in two strings which support a weight of 7 lbs. when one of the strings is horizontal, and the other is inclined at an angle to the vertical of (a) 30°, (b) 45°.

Figs. 10*a*, *b*, page 17.

Pull in cord *BA* by expt. = 8·6 – ·5 = 8·1, by drawing = 8 lbs.
„ „ „ *CB* „ „ = 4·5 – ·5 = 4·0, „ „ = 4 lbs.

Experiment VI. **A cord has its ends tied at two points in the same horizontal line, and a weight is suspended from its middle point. To find graphically and experimentally how the pull in the two parts of the cord varies with the angle between them.**

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APPARATUS. (Fig. 6.) A board with hooks, as in Fig. 6. Two spring balances reading to 8 lbs. by 2 oz., a weight of 4 lbs., a small ring, and some cord.

Method. (1) Pass the cord through the ring, and then tie each end to spring balances which hang from the hooks in the upper corner of the board, and adjust the length of the cord, so that the ring hangs some 6″ or 8″ above the lower edge. (2) Tie a 4 lb. weight to the ring. (3) Fix a piece of paper behind the cords, and adjust the length of the cord, until the angle between the two parts of the cord is less than a right angle, say about 60°. (N.B. This can be done either by shortening or lengthening the cord, or by moving in or out one of the hooks to which the balances are attached. The use of the ring ensures the weight being always at the middle of the cord.) (4) Repeat the experiment for angles between the two parts of the cord of about a right angle, and about 120°. (5) Then proceed as in Experiment V. by drawing a structure diagram and force diagram for each position of the cords, and record the result exactly as in Experiment V. (6) Write down what the experiment teaches you as to how the pull in the cord varies with the angle between the two parts of it; and reason from this what you think the pull should be (*a*) when the two parts of the cords are vertical, (*b*) when the cord is horizontal.

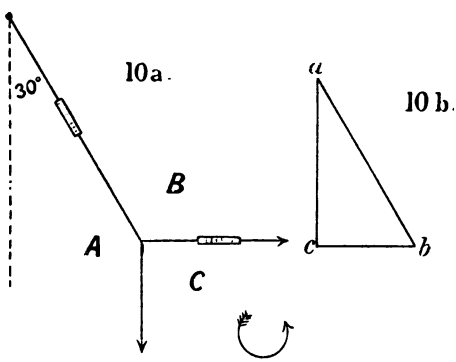

FIG. 10*a*. FIG. 10*b*.

Structure diagram. Force diagram. Scale 1″ to 2 lbs.

EXPERIMENT VII. **Take a piece of ordinary sewing cotton and a 2 lb. weight. Pass the cotton through the ring of the weight, and take one end of the cotton in each hand. Hold up the weight by the cotton, keeping the hands near**

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together, so that the two parts of the cotton are nearly parallel. Then slowly move the hands apart, so that the angle between the two parts of the cotton becomes greater and greater, and continue this until the cotton breaks, and allows the weight to fall. Explain why it is that the cotton breaks when it does; and find by construction the angle between the two parts of the cotton when it breaks, assuming that the pull required to break the cotton is 5 lbs.

EXERCISES.

The following exercises are to be worked graphically :—

21. A particle weighing 12 lbs. is kept at rest by two strings, one of which is horizontal, and the other inclined to the vertical at an angle of 60°. Find the tension in each string.

22. A weight of 24 lbs. is suspended by two flexible strings, one of which is horizontal, and the other is inclined at an angle of 30° to the vertical direction. What is the tension in each string?

23. A stone weighing 1 ton is suspended in the air by a chain ; a rope fastened to the stone is pulled so that the chain makes 30° and the rope 60° with the vertical. Draw a figure showing the three forces acting on the stone, and a triangle representing them. Find the pull on the rope.

24. A weight of 20 lbs., suspended by a string from a peg P, is pulled aside by another string knotted to the first at a point K, and pulled horizontally. Find the force necessary to pull it until PK is 60° from the vertical ; and find at the same time the force on the peg.

25. A weight of 17 lbs. is supported by two strings of lengths 15 and 18 inches respectively, whose other ends are tied to two pegs in a horizontal line, and 17 inches apart. Find the tension of each string.

26. A mass of 10 lbs. is supported by two strings inclined respectively at 45° and 30° to the horizontal plane. Find the pull of each string.

27. A string 14 feet long has its extremities fastened to two points in a horizontal line, which are 10 feet apart. A mass of 56 lbs. being attached to a point of the string 6 feet from one end, find the tension of the strings.

28. A wire is stretched horizontally between two points 6 feet apart, and a one-pound weight is then hung by a string from the middle of the wire, which is pulled down by the weight 1″ below the horizontal. Draw a figure showing the forces which act at the point of attachment of the weight to the wire, and find the pull on each of the end points to which the wire is fastened.

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29. A picture weighing 10 lbs. is supported by a cord which passes over a nail, and the two parts of which are at right angles to one another, the top of the picture being horizontal. Find the pull of either part of the string and the reaction of the nail.

30. A picture weighing 56 lbs. is slung over a nail in the ordinary way by a cord attached to two eyes in the top horizontal bar of its frame If the height of the nail above this bar is half the distance between the eyes, what is the tension in the cord? Under what circumstances would the tension be equal to or greater than the whole weight of the picture?

31. A box weighing $\frac{1}{2}$ ton has to be lifted by a crane, and a double rope or "sling" is placed round the box, with its end looping over the hook of the crane. Find the pull in the parts of the rope or sling when the angle between them is (*a*) 60°, (*b*) 90°. If the rope is weak, which is the safer position?

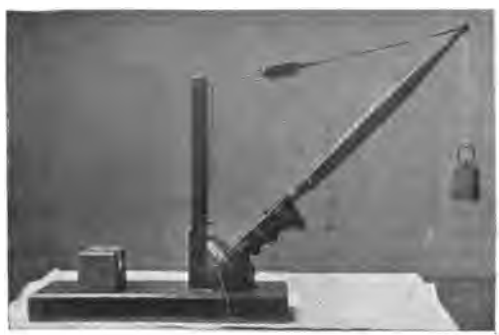

FIG. 12.

Forces acting in a Simple Crane.

EXPERIMENT VIII. **To find graphically and experimentally the forces acting in the tie and jib of a simple crane when loaded with a 4 lb. weight.**

APPARATUS. (*a*) *Portable Crane* (Fig. 12). The base board is about 24″ × 9″ × 2½″, the upright king post being about 20″ long, and 2½″ × 1½″ at the top, tapering towards the base board, as shown. An ordinary compression balance reading to 14 lbs.

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by 1 oz. is rigidly fixed by an inclined block to the base board and king post, as shown; and the wood jib, which measures about $1\frac{3}{4}'' \times \frac{3}{4}''$ at the middle, is fastened to the spindle of the balance by a simple forked joint, which allows of movement in a vertical plane. The jib is about 22″ long, and has a small light pulley screwed into its upper end. The tie of the crane consists of a light brass chain, one end of which is fastened to a wire shackle at the upper end of the jib, and the other end to a light spring balance, reading to 8 lbs. by 2 ozs. A cord is tied to the other end of the balance, and passing through one of the eyes which are screwed into the king post, is fastened by winding and tying round a hook. The angle of the tie can be altered by passing the cord through the different eyes of the king post.

FIG. 13.

(*b*) *Wall Crane* (Fig. 13). The construction of this is similar to the portable crane, and will be readily followed from the illustration. The board to which the compression balance for the jib is fixed is hinged at its upper edge to the wall board, so that the jib can be set at different angles. The compression balance reads up to 50 lbs. by 2 ozs., and the tension balance to 20 lbs. by 4 ozs., the length of the wood jib being about 22″. In the illustration the weight is shown suspended by a cord, which represents the chain of an ordinary crane; but for the purpose of the experiment now being described this is not necessary.

Method. (The following description refers particularly to the portable crane of Fig. 12, but the student will easily apply it to the wall crane.) (1) Take the reading of the balances when the crane is unloaded. (2) Suspend a 4 lb. weight from the pulley of the crane, as shown in Fig 12. (3) Adjust the length of the tie until the centre line of the jib is in the same plane as the centre of the compression balance. This is conveniently done by placing a metre or yard scale across the dial-face of the balance, and sighting the jib with one of its edges. (4) Read the two spring

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balances. NOTE.—The crane is an example of three forces acting at a point in equilibrium, the line of action of all the forces being known, and the magnitude of one of them; for the three forces are (*a*) the load on the crane, the magnitude, and line of action being known; (*b*) the force in the jib; (*c*) the force in the tie. Before we can determine these forces graphically we must draw a "structure diagram" and a "force diagram," and to draw the first of these we must take measurements from the crane. This is conveniently done as follows: (5) Tie a small weight by a piece of cotton to some point in the tie, so that it forms a plumb line, or vertical line, as shown in Fig. 12. Then measure the length of the three sides of the triangle thus formed, and draw the structure diagram to some convenient linear scale, as in Fig. 14*a*. (6) Draw the force diagram, as in Fig. 14*b*, as follows: Draw the line *a b* parallel to the line of action of the load on

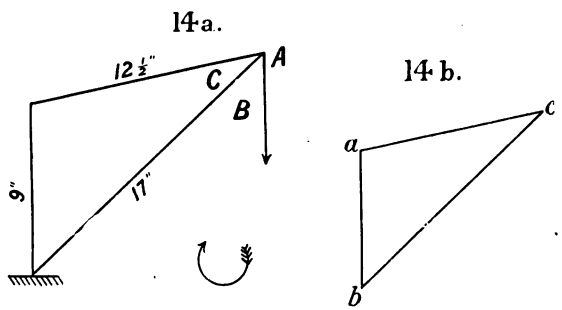

FIG. 14*a*. FIG. 14*b*.

Structure diagram. Scale ¼. Force diagram. Scale 1 cm. to 1 lb. weight.

the crane *A B*, and make its length proportional to the load to the scale chosen. From one end as *b* draw a line parallel to the jib, and from the other end a line parallel to the tie, meeting in the point *c*. Then the length of *b c* represents the force in the jib *B C*, and the length of *c a* represents the force in the tie *C A*. (7) Notice the direction of the forces in the jib and tie as follows: By making the structure diagram as in the figure, we are going round the diagram in a "clockwise" direction; therefore the load is *A B*, and we see from the force diagram it is acting from *a* to *b*, or downwards; also the force in the jib is *B C*, which from the force diagram means from *b* to *c*, or *upwards* towards the point of support; that is, the force in the jib is a *push or thrust*. Similarly the force in the tie is *C A*; that is, from *c* to *a* in the force diagram, or downwards from the point; therefore the force in the tie is a *pull*. (8) Write down the force in the tie and jib as found by drawing and by experiment, and record the results in your Laboratory note-book as follows:—

RECORD OF EXPERIMENT. *To find graphically and by experi-*

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ment the forces in the tie and jib of a crane when supporting a load of 4 lbs.

Figs. 14*a*, *b*, page 21.

Pull in tie: By experiment = 5·5 lbs.; by drawing = 5·5 lbs.
„ jib: „ „ = 7·6 lbs.; „ = 7·5 lbs.

EXERCISES.

The following exercises are to be worked graphically:—

32 A load W of 2000 lbs. is hung from a pin P, at which pieces AP and BP meet like the tie and jib of a crane. The angles WPB and WPA are 30° and 60° respectively. Show by a sketch how to find the forces in AP and BP.

33. In a crane, show the method of estimating the tension of the tie and thrust on the jib when a given weight is hung from the end of the jib If the load is 6 tons, and the tension of the tie rod (which makes an angle of 60° with the vertical) is 18 tons, find, by a diagram drawn to scale, the thrust on the jib.

34. A weightless rod 3 feet long is supported horizontally, one end being hinged to a vertical wall, and the other attached by a string to a point 4 feet above the hinge. A weight of 180 lbs. is hung from the end supported by the string. Find the pull in the string and the force in the rod.

The Inclined Plane.

EXPERIMENT IX. **To find the conditions of equilibrium of a body resting upon an inclined plane, the body and plane being supposed smooth.**

FIG. 15*a*.

APPARATUS. (Fig. 15*a*.) A board free from knots and twist about 30″ long, 7″ wide, and 1″ thick, having one of its faces covered with a sheet of glass. One end of the board is hinged to a stout base board, and the other is raised to any desired height by a hinged stay fixed in any position on the base board by a

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simple screw. The upright stay is clamped to the base board so that a small brass pulley can be clamped to it at any desired height, the groove of the pulley coinciding with the centre line of the board. A small wood box about $7\frac{1}{2}''$ by $5''$, and just large enough inside to contain a 7 lb. weight, is provided with three light easily running hard wood pulleys about $2''$ diameter.

Method. (1) Weigh the box and the scale-pan. (2) Arrange the box on the plane with a cord passing over the pulley to a scale-pan as shown in Fig. 15*a*. (3) Place a weight of 7 lbs. in the box, and add sufficient weights to the scale-pan to keep the box from moving. (4) Adjust the

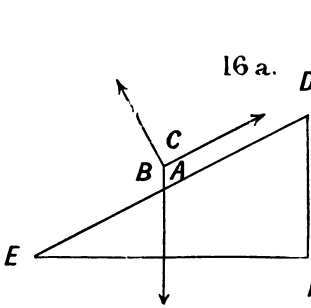

FIG. 16*a*.
STRUCTURE DIAGRAM.

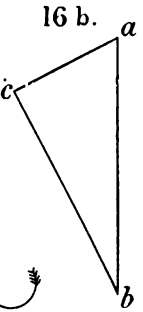

FIG. 16*b*.
FORCE DIAGRAM.

height of the pulley so that the cord is parallel to the surface of the plane. (5) Adjust the weights in the scale-pan until they are just sufficient to keep the box in a position of equilibrium. NOTE.—It is better to find the weights required in the pan to just move the box up the plane, and then take out weights until the box just moves down the plane. The mean of these is the force acquired. (6) Notice the forces acting on the box which keep it at rest; they are, (*a*) the weight of the box and contained weight acting vertically downwards; (*b*) the pull in the cord acting up the plane which is equal to the weight of the scale-pan and contained weights; (*c*) the reaction of the plane against the box which, as the plane is supposed to be smooth, acts at right angles to the surface of the plane. (7) Measure the base and height of the plane to some mark near its upper end, and draw a diagram to scale to represent the plane, and show by lines how the forces act upon the box as in Fig. 16*a*. This figure is the "structure diagram." (8) Draw a force diagram to scale as before, starting with the force *A B*, *i. e.*, the weight of the box and contained weights, and completing the diagram as in Fig. 16*b*.

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Then the line bc represents the magnitude of the reaction of the plane, and the line ca should be equal to the pull in the cord. (9) Measure and compare the angle abc of the force triangle with the angle of inclination of the plane to the horizontal; and the angle cab of the force triangle with the angle of the plane to the vertical. They should be found to be equal, that is, the triangle abc is similar to the triangle DEF. (10) The results of the experiment should be entered in the Laboratory note-book as follows:—

RECORD OF EXPERIMENT. *To find the conditions of equilibrium of a smooth body resting upon a smooth inclined plane.*

Weight of sledge = 132½ ozs. Reaction of plane by drawing = bc = 119 ozs.

Figs. 16*a*, *b*, page 23.

Pull up the plane by experiment (mean) = 60 ozs.

" " drawing = 60 ozs.

The triangle abc of the force diagram is similar to the triangle DEF of the inclined plane in the structure diagram.

What this Experiment teaches Because the force triangle abc is similar to the triangle DEF of the inclined plane, we know that the following is true:—

As $ab : bc : ca$, so is $DE : EF : FD$;

That is, as weight on plane : reaction of plane : force along plane, so is length of plane : base of plane : height of plane.

Therefore when the supporting force acts parallel to the surface or length of the plane, if the length of the plane represents the weight on the plane, then the base of the plane represents the reaction of the plane, and the height of the plane represents the supporting force.

FIG. 15*b*.

EXPERIMENT X. **To find the conditions of equilibrium of a smooth body on a smooth inclined plane when the line of action of the force maintaining equilibrium is horizontal.**

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APPARATUS. Same as for Experiment IX., with addition of the light wire frame, as shown in Fig. 15*b*.

Method. Proceed exactly as in Experiment IX., but arranging that the pull in the cord shall be horizontal by adjusting the wire frame and pulley to suit.

The student should learn from this experiment that if the weight on the plane be represented by the length of its base, then the reaction of the plane is represented by the length of the plane, and the force which maintains equilibrium by the height of the plane.

EXERCISES.

The following exercises to be worked graphically :—

35. Friction being neglected, find the force acting parallel to the plane which will support 1 ton on an incline of 1 ft. vertical and 10 ft. along the incline. If the incline were 1 ft. vertical and 280 ft. along the incline, find the force in pounds which would support 1 ton.

36. A rod AB is placed on a smooth inclined plane, and the end A is tied by a string AC to a fixed point C on the plane. Show how to find the pressure on the plane, and the tension of the string.
If the length of the base of the plane is 12 ft. and the height of the incline is 5 ft., and if the rod weighs 10 lbs., find the numerical values of the pressure and the tension.

37. AB is an inclined plane, C is a given fixed point above the plane, P is a particle of given weight at rest on the plane, and it is supported by a thread tied to C. State what forces act on P, and show how to draw a triangle for them.

38. A body weighing 187 lbs. is supported on a smooth inclined plane, whose angle is 30°, by a horizontal force. Find the force necessary to keep the body at rest.

39. A body weighing 6 lbs. is placed on a smooth plane, which is inclined at 30° to the horizon. Find the two directions in which a force equal to the weight of the body may act to produce equilibrium. Also find what is the pressure on the plane in each case.

40. Draw a vertical line AB, A above B, and draw AC making an angle of 30° with AB. Let AC represent an inclined plane, and let a particle (P) placed on it be supported by a thread which passes over A, and is tied to a fixed point (Q) in AB. Show in a diagram the forces which keep P at rest. If P weighs 20 lbs., find by construction or otherwise the forces which support P. Find also the pull on the fixed point Q, and the pressure on the top of the plane at A.

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41 A snow slope rises a height of 50 ft. in a slope of 200 ft. A sledge weighing 400 lbs. is drawn up it by a rope parallel to the surface of the snow. Find a triangle representing in magnitude the forces acting, and find the pull in the rope when the sledge is going steadily up.

EXPERIMENT XI. **To find by experiment and graphically the forces acting at the crosshead of a steam engine, and the crank effort for different positions of the crank, supposing the pressure on the piston to be constant.**

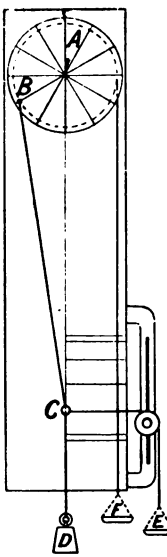

FIG. 17.

APPARATUS. (Fig. 17.) A board about 4′ long and 14″ wide is fixed with its plane vertical. *A* is a wood pulley 10″ diameter, turning on a smooth centre and represents the crank disc; it is divided into 12 equal parts by radial lines, and its edge is grooved to receive a cord. The cord *BC* is fixed to the pulley at *B*, at a point corresponding to the bottom of the groove, so that the length from *B* to the centre of the pulley represents the crank. The other end of the cord is fastened to a small ring at *C* which represents the crosshead, so that the cord *BC* is the connecting rod. A weight *D* is hung from a cord to the ring *C*, and represents the pressure on the piston. From the ring *C* a horizontal cord passes over a pulley as shown to a scale-pan *E*, which represents the pressure on the guide block. The centre, carrying the pulley, moves in the slot of a light metal arm fastened vertically to the edge of the board, and so arranged as to admit of vertical adjustment through a distance of 10″. A cord passes round the groove of the pulley to a scale-pan *F* as shown, which represents the "crank effort," that is, the component of the force along the connecting rod, which, acting tangentially to the crank, is tending to turn the crank round about the centre *A*. A pointer is fixed to the upper part of the board in order to allow the wood pulley to be turned through its different divisions; and a vertical line is drawn through the centre *A* (the centre of the shaft) to act as a guide for the cord

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CD, while horizontal lines are drawn across the board at intervals to act as a guide for the cord *CE*.

Method. (1) Tie on a weight *D*, say of 4 lbs., to represent the pressure on the piston, and let the cord *BC* coincide with the centre line, so that the connecting rod and crank are on the "bottom dead centre." It is evident that in this position, with *BC* and *CD* (the connecting rod and piston rod) in one straight line, there can be no pressure on the guide, and no crank effort. (2) Place sufficient weights in the scale-pan *F* to turn the crank disc or pulley round through, say, the first $\frac{1}{12}$th of its revolution, and place weights in the scale-pan to bring the ring *C* over the centre line. Adjust the height of the pulley for the cord *CE*, and adjust the weights in the scale-pans until the cord *CE* is horizontal, and the cord *CD* coincides with the centre line. (3) Place a sheet of paper behind the cords, and mark their positions as in previous experiments. Note the crank effort and pressure on the guide. (4) Repeat the experiment for the other positions of the crank. (5) Find graphically the components of the pressure on the piston along the connecting rod and on the guide, and then find the turning effort on the crank. This should be done for each position of the crank, and the results compared with those found by experiment. NOTE.—Of the three forces acting at the point *C* (the crosshead), the direction of all are known, and the magnitude of one of them; therefore, the magnitude of the other two can be easily determined by drawing a triangle of forces. In order to find the turning effort on the crank in any position, the pressure along the connecting rod at that position must be resolved into two components, one being tangent to the crank, and the other normal to it; the former is the crank effort. After finding the pressure on the guides and the crank effort for each of the 12 positions of the crank, they can be represented by a curve as follows: Draw any horizontal line *AB* (Fig. 17*a*), and divide it into 12 equal parts of say 1″ length, 1–2–3--6. From each point of division draw vertical lines, and along these lines set off distances to represent the pressure on the guide, and the crank effort to a suitable scale, say, of 1″ to 1 lb. for each position of the crank, 1–2–3--6. Then draw curves through the points. These curves will then represent the variation of crank effort and pressure on the guide for one revolution of the crank, with a constant pressure on the piston. Fig. 17*a* shows these curves for one stroke, or half a revolution. *C* is the crank effort curve, and *D* the curve of guide pressure.

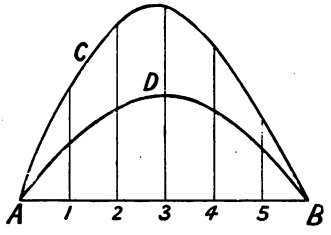

FIG. 17*a*.

What this Experiment teaches. Every student ought to be interested in this experiment, because it has to do with the

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parts of an engine, and no student ought to have performed it without finding it full of suggestions and instruction. He will see how very much the crank effort varies during a revolution, and this should set him thinking of why it is some engines have two cranks, or even three or more, and what effect this has on the combined effort turning the shaft. Then, again, the pressure on the piston of an engine is not constant, and generally gets less towards the end of the stroke. The student should reason out what effect this will have on the crank effort, and how it would alter the diagrams just drawn. In later experiments he will discover still more about these important parts of an engine.

EXERCISES.

The following exercises are to be worked graphically:—

42. A piston is 10″ diameter, and the maximum steam pressure is 90 lbs. per square inch. The stroke is 10″, and the connecting rod is 2½′ long. Find the pressure on the guide, the force along the connecting rod, and the turning effort at the crank pin when the connecting rod and crank are at right angles.

43. Draw a line diagram representing the piston rod, crosshead, connecting rod, and crank of a horizontal engine. Then show by diagrams the pressure on the guide, and the force along the connecting rod under the following conditions: (*a*) when the piston is moving from left to right; (*b*) when the piston is moving from right to left. State whether the pressure is on the same guide in each case.

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CHAPTER II.

EQUILIBRIUM OF MORE THAN THREE FORCES ACTING AT A POINT. POLYGON OF FORCES. CRANE WITH CHAIN. SIMPLE ROOF TRUSS.

EXPERIMENT I. **To find the conditions of equilibrium of five forces acting at a point.**

APPARATUS. [1]A board similar to that used for Experiment I., Chapter I., but with four pulleys and five cords, as in Fig. 18.

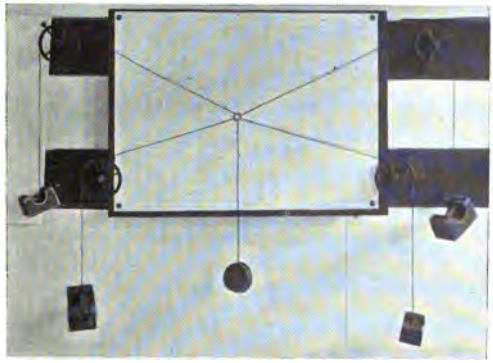

FIG. 18.

[1] A vertical board with four balances and a weight very similar to Fig. 6 may be used, or a horizontal board with five balances, similar to Fig. 7.

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Method. (1) Attach unequal weights to the ends of the five cords, so that the ring assumes a convenient position of equilibrium, as shown in Fig. 18. (2) Place a sheet of paper behind the cords, and mark points coinciding with their position and the centre of the ring exactly as for former experiments. Also write on the paper the magnitude of the weights on the cords, and show their direction by arrows. (3) Remove the paper, and draw lines from the point corresponding to the centre of the ring, to represent the line of action of the five forces, and mark the spaces, as shown in Fig. 19*a*. This figure is the structure diagram. (4) Draw the force diagram to scale as follows: (Fig. 19*b*) draw a line $a\,b$ parallel to the cord $A\,B$, and to represent the pull in this cord. From the end b draw a line $b\,c$ parallel to the cord $B\,C$, and to represent the pull in $B\,C$. Continue in this way for the other cords $C\,D$, $D\,E$, and $E\,A$. The last line drawn

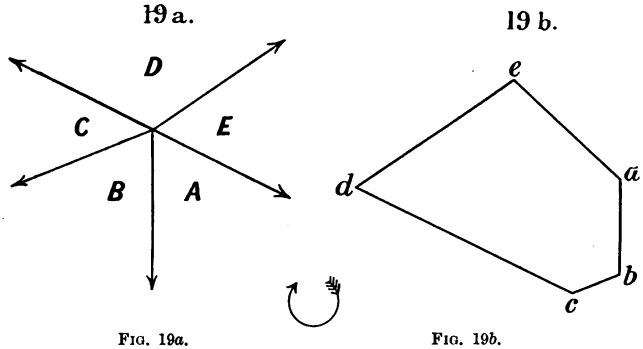

FIG. 19*a*. Structure diagram.

FIG. 19*b*. Force diagram. Scale 1″ to 1 lb. weight.

$e\,a$ should meet the first line drawn $a\,b$ at the end a, thus forming a closed polygon. But owing to the use of the pulleys and inaccuracies in working the line $e\,a$ may not meet the line $a\,b$ at the end a. Its line of action may not coincide with the actual line which joins the points e and a, and closes the polygon. In such a case it is better to draw a line $e\,a$ parallel to the cord $E\,A$, and equal to the pull in the cord, and then to draw a dotted line which actually closes the polygon. The difference in position and magnitude of these two lines will then represent the error of the experiment.

RECORD OF EXPERIMENT. *Experiment on the conditions of equilibrium of five forces of* 1, 2, 3, 4, *and* 5 *lbs. weight acting at a point.*

Figs. 19*a*, *b*.

The force diagram, each side of which is respectively parallel and proportional to one of the forces, is a closed

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polygon. The last line drawn $e\,a$ measures 1·48″, and should measure 1·5″. It is almost exactly parallel to the force line $E\,A$.

What the Experiment teaches. Allowing for slight and very reasonable inaccuracies in working the experiment, it would appear that a conclusion which may be fairly drawn from the above experiment is, that lines drawn respectively equal and parallel to the forces form a closed polygon. The proof that this is actually the case is a well-known part of mechanics, and is expressed by what is known as the principle of the polygon of forces, the meaning of which the student should now be able to understand and appreciate, and which may be stated as follows:—

Polygon of Forces. *If a number of forces acting at a point can be represented in magnitude and direction by the sides of a closed polygon taken the same way round, the forces are in equilibrium;* and conversely,

If any number of forces acting at a point be in equilibrium, and lines be drawn parallel to and representing the magnitude of the forces respectively taken the same way round, then these lines will form a closed polygon.

[1]By referring to Figs. 19*a*, 19*b*, we see that if any one of the five forces is removed there will no longer be equilibrium; therefore any one of the five forces is the equilibrant of the other four. But if the direction of the equilibrant be reversed, it becomes the resultant; therefore by reversing the direction of any one of the five forces, it becomes the resultant of the other four. From this it should be clear to the student that if the polygon of forces does not form a closed figure, the closing line represents a force which is the equilibrant if it acts the same way round as the other forces, and the resultant if it acts the opposite way round. Hence the polygon of forces not only enables us to determine whether a certain number of forces are in equilibrium, but it enables us to find the equilibrant and the resultant of any number of given forces which are not in equilibrium.

[1] The student will learn later that the proposition is also true for forces not acting at a point.

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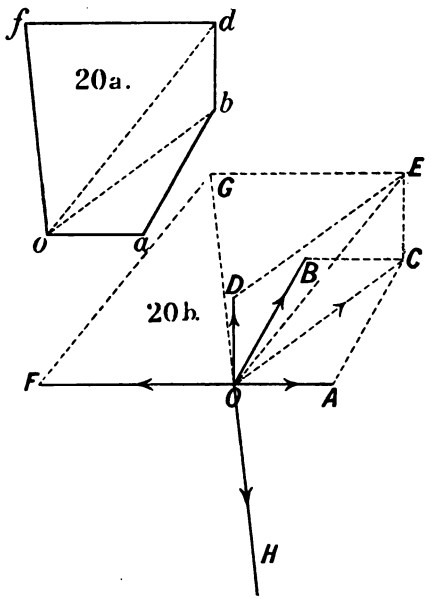
Another proof of the Polygon of Forces. *An exercise in drawing.* A proof of the polygon of forces may be obtained by using the principle of the parallelogram of forces, as follows:

Mark any point O on the paper (Fig. 20*b*), and draw two lines OA, OB to represent forces of 2lbs. and 3lbs. respectively acting at any angle, of say 60°. (Scale 1″ to 1lb.) Find the resultant OC of these two forces by the parallelogram of forces as shown by the dotted lines of the figure Draw a third line OD to represent a force of $1\frac{3}{4}$lbs. acting vertically upwards, and find by the parallelogram of forces the resultant OE of this force and the force OC. Notice that OE is the resultant of the three forces OA, OB, OD. Draw a line OF to represent a fourth force of $2\frac{1}{2}$ lbs. acting in the same line as OA, and find as before the resultant OG of this force and of the force OE. Then OG is the resultant of the four forces OA, OB, OD, and OF, and a line OH drawn as shown in the figure, equal and opposite to the line OG, represents the equilibrant of these four forces. Therefore the five forces OA, OB, OD, OF, OH are in equilibrium. Now draw a force diagram (Fig. 20*a*) having its sides respectively proportional and parallel to the five forces OA, OB, OD, OF, and OH, taking them the same way round; thus oa is parallel to and represents OA, ab is parallel to and represents OB, and so on. The figure forms a closed polygon,



FIGS. 20*a*, 20*b*.

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which is in accordance with the statement of the proposition of the polygon of forces which we wished to verify. Notice also that in drawing the polygon $oabdf$ we have arrived at the same result as by the continued application of the parallelogram of forces, but in a much simpler way. For join the point o to the points b and d. Then observe that the triangle oab Fig. 20*a* is similar to the triangle OAC of Fig. 20*b*, which is half the parallelogram drawn to obtain the resultant of the forces OA and OB. Therefore ob in Fig. 20*a* coincides with OC in Fig. 20*b*, and is the resultant of the two forces represented by the lines oa, ob. Also od in Fig. 20*a* coincides with OE in Fig. 20*b*, and is the resultant of forces represented by the lines ab, bd; that is, of forces represented by the lines oa, ab, bd. Similarly the line of in Fig. 20*a* is the resultant of the forces represented by the lines od, df; that is, of four forces represented by the lines oa, ab, bd, df. Therefore the line fo of Fig. 20*a*, which is the closing line of the force polygon, is the equilibrant of the other four forces.

EXERCISES.

The following exercises are to be worked graphically. Example No. 1 should be worked by every student before proceeding to others.

1. The lines joining the centre of a regular hexagon to its angular points represent the lines of action of six forces of $1\frac{1}{2}$, 2, $2\frac{1}{2}$, 3, $3\frac{1}{2}$, and 4 lbs. weight, all of which act outwards from the centre. Find the resultant of these forces in magnitude and direction, by drawing three force polygons as follows: (*a*) taking the forces in the order named; (*b*) taking the forces in the following order: 2, 3, 4, $1\frac{1}{2}$, $2\frac{1}{2}$, $3\frac{1}{2}$ lbs.; (*c*) taking the forces in the following order: $1\frac{1}{2}$, $2\frac{1}{2}$, 2, $3\frac{1}{2}$, 3, 4. Write down the conclusion you draw from the exercise.

2. Forces of 1, 2, 3, 4, 5, 6, 7, and 8 lbs., acting at a point, make equal angles with one another. Find their resultant.

3. Forces of 9, 10, and 12 lbs. act at a point. The angle between the 9 lb. and 10 lb. forces is 30°, and the angle between the 9 lb. and 12 lb. forces measured in the same direction is 135°. Find the resultant of the forces.

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Crane with Chain.

EXPERIMENT II. **To find the forces acting in the tie and jib of a simple crane when the load is suspended from a chain.**

APPARATUS. The portable crane and wall crane as used for Experiment VIII., Chapter I., except that the load is suspended by a cord representing the chain of an ordinary crane, which passes over the pulley at the end of the jib, and is fastened at some point in the king post between the jib and tie.

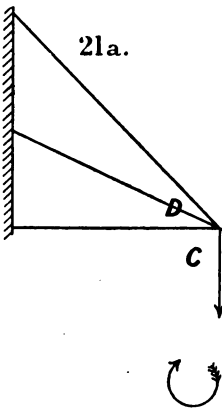

FIG. 21*a*.
STRUCTURE DIAGRAM.
Scale $\frac{1}{8}$.

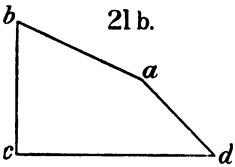

FIG. 21*b*.
FORCE DIAGRAM.
Scale 1 cm. to 2 lbs

Method. The wall crane, Fig. 13, with the jib horizontal. (1) Arrange the load of 14 lbs so that the chain is about midway between the jib and tie, and adjust the tie so that the jib is horizontal. (2) Fasten a small plumb line as before, and take measurements for drawing the structure diagram. (3) Notice that the crane is now an example of four forces acting at a point as follows: (*a*) the pull of the load acting vertically downwards; (*b*) the pull in the part of the chain between the jib and tie which, neglecting any loss due to passing the chain over the pulley, is equal to the load on the crane; (*c*) the push in the jib; (*d*) the pull in the tie. The lines of action of all four are known, and the magnitude of the first two. (4) Draw the structure diagram to a linear scale, as in Fig. 21*a*, and letter the spaces as shown. (5) Draw the force diagram to scale, as in Fig. 21*b*, and enter the results of the experiment as before.

RECORD OF EXPERIMENT. *To find the forces in the jib and tie of a wall crane when using a chain, and when loaded with* 14 *lbs.*

Figs. 21*a*, *b*.

Push in jib by experiment = $21\frac{3}{8}$ lbs., by drawing = $21\frac{1}{4}$ lbs.
Pull in tie „ = $11\frac{5}{8}$ „ „ = $11\frac{1}{2}$ lbs.

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Simple Roof Truss.

EXPERIMENT III. **To find the forces acting in the different members of a simple triangular roof truss.**

APPARATUS. (Fig. 22.) The roof truss consists of two strips of wood, each 3 feet long and $1\frac{1}{4}'' \times \frac{1}{2}''$, fastened together at the upper end by a loosely-fitting bolt and nut, the lower ends being held together by a stout cord, which acts as a tie, and the angle of slope being about 30°. A spring balance, arranged as shown in the illustration, enables the pull in the tie to be measured. The ends of the truss rest on two supports, and a 28 lb weight is hung from the upper joint. The tie can be shortened to give any desired span and pitch of roof.

FIG. 22.

Method. (1) Read the spring balance before loading the truss. (2) Hang on the load, and take measurements for the structure diagram. (3) Draw the structure diagram, Fig. 23*a*, as before. (4) Draw the force diagram as follows: Consider first the forces acting at the left-hand bottom corner of the roof, the joint *A B D*. These are three in number, viz.: (*a*) the reaction of the support acting vertically upwards (*A B*), and equal to half the load on the truss, or 14 lbs, since the load is central (this is neglecting the weight of the truss itself); (*b*) the force in the sloping rafter (*B D*); (*c*) the force in the horizontal tie (*D A*). The directions of all three are known, and the magnitude of the first. Therefore draw the triangle of forces for this joint as shown in Fig. 23*b*. Next consider the top joint, the joint *D B C*. Here there are also three forces, viz: (*a*) the load of 28 lbs. (*B C*); (*b*) the force in the sloping rafters (*D B*) and (*C D*). The directions of all three and the magnitude of the first are known. Draw then its force diagram as shown in Fig. 23*c*. Then consider the right-hand bottom joint *C A D*, the forces at which are similar to those at the left-hand joint, and draw its force diagram, as in Fig. 23*d*. Now

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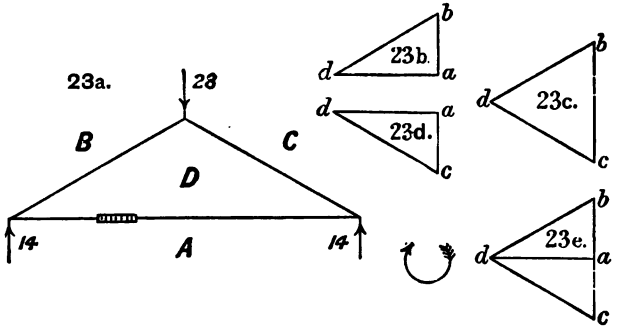
notice that the line $b\,d$ occurs in the two figures 23*b* and *c*; that the line $d\,c$ occurs in the two figures 23*c* and *d*; and that in fact two of the three lines in any one of the figures occur also in the other two figures. There can be no good reason why the same line should be drawn twice; and the student will readily see how the three diagrams can be combined in one, as in Fig. 23*e*, where each line occurs once only, and which gives us exactly the same information as the three separate diagrams. He will see that although a force triangle can be drawn for each joint, yet a single force polygon can be drawn for the whole truss.

To find whether the parts of the truss are ties or struts.

One example will be sufficient, say that of the joint $A\,B\,D$. The cycle is clockwise, therefore the sloping rafter is $B\,D$. Looking at the force



FIGS. 23*a*, *b*, *c*, *d*, *e*.

diagram, Fig. 23*b*, we see that $b\,d$ means that the force acts from *b* to *d*, and therefore acts downwards in the rafter, *pushing* at the joint $A\,B\,D$. The rafter is therefore a *strut*, as only a strut can exert a push. Also the horizontal member at the same joint is $D\,A$, which, referred to the force diagram, means that the force acts from *d* to *a*; and therefore it *pulls* away from the joint, so that the member $D\,A$ is a *tie*. The fact of its being a cord shows that it cannot be a strut; but we do not, of course, have cords in actual roofs. The student should apply the same method to the other joints.

This is only a simple example, but the student's interest in it should be increased by knowing that precisely the same method applies for any roof or bridge truss, however complicated. In order to distinguish struts from ties in the structure diagram, and tensile from compressive forces in the force diagram, it is usual to show ties and tensile force lines by thin lines, and struts and compressive force lines by thick lines.

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RECORD OF EXPERIMENT. *Experiment on the forces acting in the members of a triangular roof truss when loaded with* 28 *lbs.*

Figs. 23*a*, *e*, page 36.

Push in struts $B\,D$ and $D\,C$ = 28 lbs.

Pull in tie $D\,A$ by drawing = 24 2 lbs., by experiment = 24 lbs. 3 ozs.

EXERCISES.

The following exercises are to be worked graphically :—

4. A beam is 16 ft. long, and has a concentrated load upon it of 8 tons in the centre. The beam is strengthened by a vertical strut 8 ft. long placed directly under the load, and the end of the strut is tied to the ends of the beam. Find the nature and amount of the stress in each portion of the structure.

5. The frame of a vertical truss is made up of three equilateral triangles, giving three lower joints and two upper joints. The truss is supported at the end lower joints, and a load of 3 tons is placed on each of the upper joints. Find the nature and amount of the stress in each bar.

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CHAPTER III.

EQUILIBRIUM OF FORCES NOT MEETING IN A POINT. FUNICULAR POLYGON. PARALLEL FORCES.

EXPERIMENT I. **To find the conditions of equilibrium when three forces not meeting in a point act upon a body in equilibrium.**

APPARATUS. (Fig. 24.) The board with pulleys as used for Experiment I., Chapter I. An irregular-shaped piece of card or thin wood about 7″ × 5″, with three holes pierced near the edge at unequal distances apart. A piece of cord is tied in each hole, and is used for attaching the weights.

FIG. 24.

Method. (1) Place the cords in position as in the illustration, and tie on weights of 4, 3, and 2 lbs., so that the cord assumes some such position as shown in Fig. 24. (2) Place a sheet of paper behind the cords, and mark the position of each force cord. (3) Remove the paper, and draw lines representing the lines of action of the forces. (4) Produce these lines, and notice if they meet in a point. (Fig. 24*a*.) (5) Draw a force diagram, Fig. 24*b*, making the length of the line $a\,b$ to represent the magnitude of the force $A\,B$. Draw $b\,c$ parallel to $B\,C$, and $c\,a$ parallel to $C\,A$. Measure $b\,c$ and $c\,a$, and see if they represent the forces $B\,C$ and $C\,A$.

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RECORD OF EXPERIMENT. *Conditions of equilibrium of three forces acting upon a body in equilibrium.*

Figs. 24*a*, *b*. The lines of action of the three forces meet in a point when produced.

$a\,b = 4''$, and represents force $A\,B$.
$b\,c = 1{\cdot}98''$ „ „ $B\,C$ of 1·98 lbs., should be 2 lbs.
$c\,a = 2{\cdot}96''$ „ „ $C\,A$ of 2·96 lbs., should be 3 lbs.

What the Experiment teaches. If the results are carefully obtained, they ought to show clearly two things; first, that the lines of action of the forces do meet in a point if produced far enough; and secondly, that the force diagram is a triangle whose sides represent the forces, exactly as with the ordinary triangle of forces. These are important laws of statics, and may be expressed as follows:—

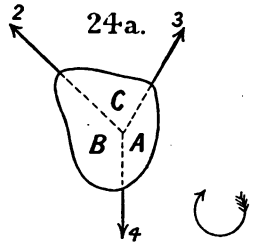

FIG. 24*a*.

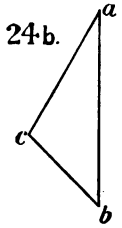

FIG. 24*b*.

Equilibrium of Three Forces. *If three forces not parallel to one another acting in one plane upon a body keep it in equilibrium, the lines of action of the forces will meet in a point if produced far enough, and lines drawn parallel to the lines of action of the forces will form a triangle, the sides of which are proportional to the magnitude of the forces to whose lines of action they are respectively parallel.*

EXERCISES.

The following exercises are to be worked graphically:—

NOTE.—In working the following examples it is necessary to remember that when a body is in contact with a smooth surface the reaction of the surface is in a direction normal to it. For example, in Exercise 1 the reaction of the plane is perpendicular to its surface.

1. Draw a horizontal line $C\,D$, take a point A vertically over C, and another point B between C and D; join $A\,B$. Suppose that $A\,B$ represents

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a uniform rod that can turn freely round a hinge at A, and rests with the end B against a smooth horizontal plane (CD), name the forces which keep the rod at rest, and show in a diagram how they act. If the weight of the rod be 10, find the numerical values of the other forces.

2. Mention the points that go to the complete specification of a force.

Draw an equilateral triangle ABC, with the side BC vertical, and C above B. Let it represent a triangular board weighing 5 lbs., capable of moving in a vertical plane round a hinge at A, but kept in position by leaning against a fixed point which touches BC at G. Find the reaction of the fixed point and of the hinge.

Specify completely the reaction of the hinge.

3. In what direction does a smooth plane exert a reaction on a body which is pressed against it?

One end of a string is fastened to a fixed point P, and the other to one end B of a rod AB. The lengths are such that AB comes to rest in an inclined position with the end A on a smooth horizontal plane below P. Explain why the position must be such that the string shall be vertical.

4. A uniform ladder 26 feet long and weighing 100 lbs. rests with its upper end against a smooth vertical wall, and is inclined at 60° to the ground. Find the pressure on the wall and ground.

5. State the conditions necessary so that a given system of forces acting on a body in one plane may be in equilibrium.

A heavy uniform rod is supported at one end by a hinge, and at the other by a string which is fastened to a point vertically above the hinge, and at a distance from it equal to the length of the rod. The rod then hangs down at an angle of 60° with the vertical. Draw a figure showing the direction of the forces acting on the rod, and calculate the ratios of their magnitude

6. Show that if three forces acting in a plane on a body keep it in equilibrium, their directions must either be parallel or pass through a point. A picture frame weighing 12 lbs. is hung by a cord passing over a nail. The two parts of the cord make an angle of 120° with each other. Find the tension in the cord.

7. A rectangular trap-door measuring 4 feet square, and weighing 75 lbs., is hinged with one edge horizontal, and is supported in the horizontal position by a chain which is connected with the middle point of the outer edge of the trap-door, and with a point vertically over the middle point of the edge in which the hinges are fixed, but 7 feet above it. Sketch the arrangement, and determine the tension upon the chain and the reaction on the hinges.

8. Draw a triangle ABC, AC being horizontal, and B below AC, and let the angles A and C be acute. Suppose AB to represent a rod of uniform density, that can turn freely round a hinge at A, and BC to represent a string fastened to B and to a fixed point at C. Name the forces which keep AB at rest, and show in a diagram how they act.

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Take G the middle point of AB, and through it draw a vertical line; produce CB to meet the vertical line in P, and through A draw a line parallel to BC to meet the vertical line in Q. Show that APQ is a triangle for the forces that keep the rod at rest.

9. A symmetrical pair of steps, hinged together at the top, and connected together by a string at the bottom, stands on a smooth horizontal plane. If the length of each side be 3 ft 3 in., and the string be 3 ft. in length, find the tension of the string when a person of 140 lbs. in weight stands on the steps at a height of 2 ft. from the ground. How is the tension of the string affected as the person ascends the steps?

10. A ladder 10 ft. long, and weighing 25 lbs, passes over a smooth wall, and projects 2 ft. beyond the top of the wall. The ladder makes an angle of 60° with the ground, and its lower end is fixed. Find the pressure on the wall when a man weighing 150 lbs. stands on the ladder at a distance of 5 ft. from the fixed end.

11. A pole 12 ft. long, weighing 25 lbs., rests with one end against the foot of a wall, and from a point of 2 ft. from the other end a cord runs horizontally to a point in the wall 8 ft. from the ground. Find the tension of the cord, and the pressure of the lower end of the pole.

EXPERIMENT II. **To find whether the proposition of the Polygon of Forces applies to forces acting upon a rigid body in one plane, which do not meet in a point.**

APPARATUS. The pulley board as used for Experiment I., Chapter II. An irregular piece of card with five cords, four of the cords to pass over pulleys with weights at each end, and the fifth cord to hang vertically. The arrangement will be similar to that for the last experiment, except that the card is acted upon by five forces instead of three.

Method. (1) Arrange the card and cords with any five convenient weights; then proceed exactly as in former experiments, and find whether the forces form a closed polygon, the sides of which are respectively parallel and proportional to the forces.

What the Experiment teaches. The student will find from this experiment that the Polygon of Forces applies equally to forces which do not act at a point, and that all that is true for forces acting at a point is also true for forces not acting at a point, provided the forces act in the same plane and are in equilibrium. Any one of the five forces is the equilibrant of the other four, and is therefore equal and opposite to the

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resultant of the other four. Hence we can find the magnitude of the resultant of any number of forces acting upon a body in one plane by drawing a Polygon of Forces, when the closing line will represent the magnitude of the resultant.

The conditions hold whatever be the position of the forces, and is therefore true for *parallel forces*. An example of this will be seen in the next experiment

FIG. 25.

EXPERIMENT III. **A wood rod is suspended at its ends in a horizontal position by two spring balances. From the points of suspension the ends of a cord are tied, and weights are suspended from any two points in the cord. To find the pull in the different parts of the cord, and the pull on the supports.**

APPARATUS. (Figs. 25 and 26*a*.) A light rigid rod pq about

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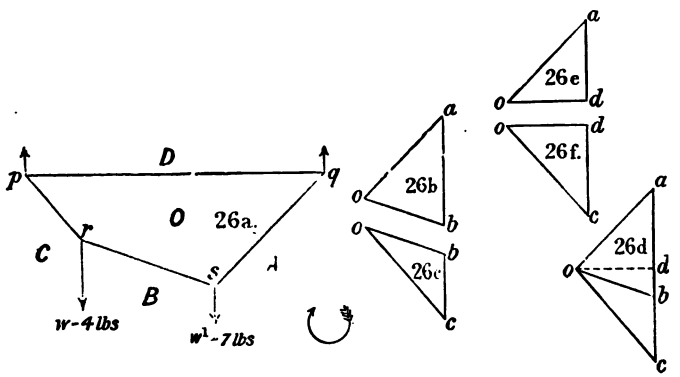
31″ long is supported at each end by spring balances reading to 8 lbs by 2 ozs. *prsq* is a cord tied to *p* and *q* with spring balances (reading to 8 and 10 lbs. by 2 ozs.) in each part. *W* and W^1 are weights of 4 and 7 lbs. respectively tied to points *r* and *s* in the cord. The whole is arranged on a vertical board, so that the rod, balances and cord hang free of the board.

Method. (1) Place a sheet of paper behind cords, and mark direction of the cords and the rod *pq*. (2) Draw the structure diagram to suitable scale as shown in Fig. 26*a*. (3) At each of the points *r* and *s* there are three forces acting, viz., the weight and the pull in the cord on each side of the point of suspension. The triangle of forces can therefore be applied



FIGS. 26*a*, *b*, *c*, *d*, *e*, *f*.

to each point, in order to find the pull in the cords, knowing the magnitude and direction of the weight.

Draw the force triangle for the point *s*, as in Fig 26*b*.
,, ,, ,, ,, *r* ,, ,, 26*c*.

Then in Fig. 26*b*, *oa* represents magnitude of pull in cord *OA*.
,, ,, *bo* ,, ,, ,, *BO*.
And in Fig. 26*c*, *ob* ,, ,, ,, *OB*.
,, ,, *co* ,, ,, ,, *CO*.

(4) Test this by measuring lengths of these lines to the scale used, and comparing with the reading of spring balances, *allowing for reading of balances when weights W and* W^1 *are off*. Note that the line *bo* occurs in each of the triangles Figs. 26*b* and 26*c*. It should evidently be the same length in each, as it represents the pull in cord *BO*.

Therefore the two triangles Figs. 26*b*, *c*, can be combined in one figure to save drawing the same line *bo* twice over. This is shown in Fig. 26*d*,

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where *ab* and *bc* form a continuous straight line, and the line *bo* is common to both triangles.

As already stated, the arrangement of Fig. 26*d* is the usual and better one to adopt. Note that the same construction will apply whatever be the number of weights tied to the cord. Their lines of action need not be parallel.

Note that the construction allows of finding the direction of the middle cord *BO*, if the direction of the end cords and the weights W and W^1 are known. (If in Fig 26*d* *ao* and *co* are drawn meeting in *O*, then line joining *b* to *o* is parallel to the middle cord *BO*.)

The figure *prsqp* is called a "funicular polygon," the line *pq* being its "closing line." Fig. 26*d* is a "polar diagram," the point *O* is the "pole," and the lines *oa*, *ob*, *oc*, are called "rays." The line *abc* is the "force or load line," or "force polygon."

(5) There are three forces acting at each of the points *p* and *q*, viz., the upward pull of the supporting balance, the pull in the end cord (*OA* or *CO*), and a force along the rod. The lines of action of all three are known, and the magnitude of the pull in the end cord. Draw then a force triangle for each point, obtaining the magnitude of the pull in the end cords from Figs. 26*b*, *c*. These are shown in Figs. 26*e* and *f*, which are drawn to the same scale as Figs. 26*b*, *c*, *d*.

Thus, in Fig. 26*e*, *ao* represents the pull in string *AO* (obtained from Fig. 26*b*), and *od* and *da* are drawn parallel to rod *OD*, and to line of the pull of balance *DA* respectively, to meet in point *d*. Similarly for Fig. 26*f*. (Note that the line *od* is the same length in each figure.) Therefore *da* (Fig 26*e*) represents the upward pull of balance at end *q*, and *cd* (Fig. 26*f*) represents upward pull of balance at end *p*. (6) Test this by measuring lengths of *da* and *cd*, and comparing with reading of balances, allowing for weight of rod. (7) Now the upward pull at *p* added to upward pull at *q* must equal $W + W^1$. Therefore, length of *da* (Fig 26*e*) added to *cd* (Fig. 26*f*) must equal *ac* (Fig. 26*d*).

(8) From pole *O*, in Fig. 26*d*, draw a line parallel to the rod *OD* (Fig. 26*a*) to meet the force line in the point *d*. (This line is shown dotted.)

Now notice that triangle *aod* (Fig. 26*e*) is equal to triangle *aod* (Fig. 26*d*).

Also ,, ,, *cod* (Fig. 26*f*) ,, ,, *cod* (,,).

Therefore, Figs. 26*e* and *f* are unnecessary, for by drawing the single line *od* in Fig. 26*d* we obtain the same result.

Therefore in Fig. 26*d* the length of *cd* represents upward pull at end *p*.

and ,, *da* ,, ,, ,, *q*.

That is, a line drawn from the pole *O* of the polar diagram parallel to the closing line of the funicular polygon, divides the force line into two parts, which represent the upward pulls at the ends of the rod.

The diagram of Fig. 26*d* therefore enables us to find—

(1) The magnitude of the pulls in each of the cords if we know the magnitude of the weights and direction of the two end cords or of all the cords.

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(2) The position or direction of the cords when the directions of any two are given.

(3) The magnitude of the upward pulls at the points of suspension of the cords.

NOTE.—The four external forces (Fig. 26*a*), viz., the weights W and W^1 and the upward pulls of the two balances are in equilibrium, and therefore form a closed polygon. As the forces are parallel, this polygon becomes a straight line. In Fig 26*d* it is made up of the four lines *ab*, *bc*, *cd*, *da*. The weights W and W^1 may act at any point in their line of action without disturbing the conditions of equilibrium; therefore they might hang direct from the rod *pq*. This should be remembered as leading up to the next experiment.

RECORD OF EXPERIMENT. *To find the pull in the different parts of a cord when supporting weights of* 4 *and* 7 *lbs., and to find the pull on the supporting balances when the arrangement is as in Fig.* 25.

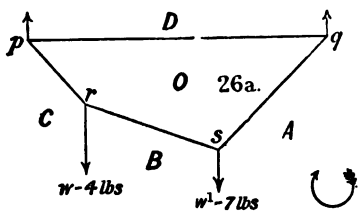

FIG. 26*a*.
STRUCTURE DIAGRAM.
Scale $\frac{1}{8}$.

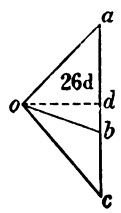

FIG. 26*d*.
FORCE DIAGRAM.
Scale 1 cm. to 1 lb.

Pull in String.		By Expt.		By Drawing.
a o	=	$7\frac{1}{4}$	=	7·2 lbs.
b o	=	$5\frac{1}{2}$	=	5·4 ,,
c o	=	$7\frac{3}{4}$	=	7·8 ,,

Pull in balance *CD* at end *p* by drawing
= 5 75, by experiment = 5·8 lbs.

Pull in balance *DA* at end *q* by drawing
= 5·25, by experiment = 5·2 lbs.

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What the Experiment teaches. This experiment illustrates some of the most important principles of graphic statics, and for that reason should be thoroughly understood by the student. If it has been carefully worked the results ought to be accurate enough to lead the student to admit that the methods adopted are reliable and capable of wide application. For example, the outline of the funicular polygon *prsq* (Fig. 26*a*) is really the shape which a string will assume when loaded with the given weights; also, if the length of the string is altered, the shape of the funicular polygon will alter, and with it the form of the

FIG. 27.

polar diagram and the position of the pole. Hence, by reversing the method we ought to be able to find the shape which the string would assume if we knew the lines of action and the magnitude of the suspended weights. Thus, in Fig. 26*d*, if the pole *O* be assumed in the same position as shown, and the lines *oa*, *ob*, and *oc* be drawn, then by drawing lines parallel to these across the lines of action of the forces in the proper order, we should obtain the funicular polygon *prsq*. The method for finding the magnitude of the supporting forces at the end of the rod is exceedingly important, and should be carefully noted.

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EXPERIMENT IV. **To determine the reactions at the supports of a horizontal beam when loaded with weights of 14 and 7 lbs. at points between the supports.**

APPARATUS. (Fig. 27.) The wood beam is 60″ long, and 2″ × 1″, the top edge being marked in lengths of 1″. The circular spring balances at the end are graduated to 40 lbs. by 2 ozs., and read the forces required to support the beam, which are usually termed the *reactions at the supports*. Weights are tied to the beam in any desired positions, and either of the balances can be raised or lowered to bring the beam horizontal.

Method. (1) Draw a structure diagram, as in Fig. 27*b*, to scale to represent the beam, and draw the lines of action of the upward pulls at the ends, and of the weights W^2 and W^3. PQ represents the beam. (2) Draw the force line *abc*, Fig. 27*c*, where *ab* represents W^2, and *bc* represents W^3. Take a pole *O* in any convenient position, and join *oa*, *ob*, *oc*. Start from any point in the line of action of the upward pull at *P*, and draw the funicular polygon as shown; thus *AO* (Fig. 27*b*) is parallel to *ao* (Fig. 27*c*). *BO* (Fig. 27*b*) is parallel to *bo* (Fig. 27*c*), and so on. (3) Draw the closing line *OD*, and from the pole *O* (Fig. 27*c*) draw *od* parallel to this line. Then *cd* represents the upward pull at the end *Q*, and *da* represents upward pull at end *P*. Compare these with reading of spring balances, allowing for weight of beam.

NOTE.—The construction should be repeated for some other position of the pole *O*.

RECORD OF EXPERIMENT. *Experiment to find the reactions at the supports of a horizontal beam when loaded with weights of 7 and 14 lbs.*

Figs. 27*b*, *c*, page 48.

c d represents reaction at *Q*
= 10·76 lbs., by experiment = (12 lbs. 4 ozs.) − (1 lb. 8 ozs.)
= 10·75 lbs.

d a represents reaction at *P*
= 10·3 lbs., by experiment (11 lbs. 14 ozs.) − (1 lb. 6 ozs.)
= 10·5 lbs.

FURTHER RESULTS FROM THIS EXPERIMENT. Consider the three forces acting at the point *h* (Fig. 27*b*). Any one of the three is equal to the resultant of the other two. Therefore a force equal to the pull in the cord *OA* acting along the line *h n*, from *h* to *n*, would be the resultant of the pull in the cord *BO* (which is represented by *b o* in Fig. 27*c*), and of W^2 (which is

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represented by $a\,b$ in Fig. 27*c*). This resultant is represented by the line $a\,o$ in Fig. 27*c*. Similarly a force represented by the line $O\,C$ in Fig. 27*c*, if it acted in the line $m\,n$ from m to n in Fig. 27*b*, would be the resultant of the pull in the cord $C\,O$ and of W^3. These two resultants meet in the point n. They can be replaced by a single force acting through this point. Now in Fig. 27*c* the line $a\,o$ represents one resultant, and the line $o\,c$ the other resultant. Therefore the line $a\,c$ represents a force equal to the resultant of these two if it acts from a to c. But $a\,c$ represents W^2 and W^3. Therefore the point n in

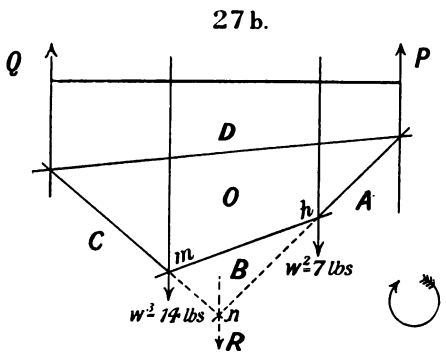

FIG. 27*b*.
STRUCTURE DIAGRAM.
Scale $\frac{1}{10}$.

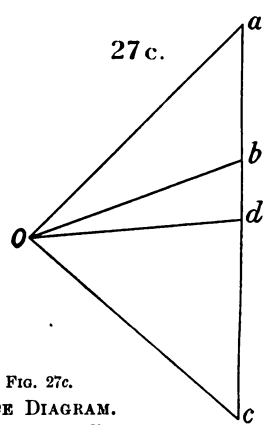

FIG. 27*c*.
FORCE DIAGRAM.
Scale 1 cm. = 2 lbs.

Fig. 27*b* is a point in the line of action of the resultant R of the weights W^2 and W^3, and its line of action is vertical, being parallel to $a\,c$ (Fig. 27*c*). Therefore the resultant of W^2 and W^3 has been found in magnitude and direction; that is, the funicular polygon, combined with a polar and force diagram, enables us to find the position of the resultant of a number of forces when their magnitude and lines of action are known.

EXPERIMENT V. **To determine the reactions at the supports of a beam when loaded with weights of 7 lbs. and 14 lbs.**

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between the supports, and when the reactions at the supports are inclined to the beam.

APPARATUS. Use the beam and balance of Experiment IV. After hanging on the weights, attach a spring balance to a screw eye in one end of the beam, and arranging this balance to be horizontal, pull the beam in a horizontal direction until the centre line of the balances showing the reactions make an angle of about 60° with the beam. Fix the horizontal balance so that its reading represents the horizontal pull.

Method. Proceed exactly as for Experiment IV., noting that the complete force polygon will no longer be a straight line, but will form a triangle representing the five forces acting on the beam, viz., the two weights, the two inclined reactions, and the horizontal end pull.

EXPERIMENT VI. **To find the magnitude and position of the resultant of a number of parallel forces.**

APPARATUS. A wood beam with cone centres, as used for experiments on moments and levers. See Fig. 29. Weights: It is necessary to use the lever in order to attach the weights, and its weight should be regarded as one of the forces.

Method. (1) Weigh the lever, and regard its weight as one of the parallel forces acting through the centre of the lever. (2) Tie a weight of 7 lbs. on one side of the lever at a distance of 12″ from the point of support. (3) Tie a weight of 4 lbs. on the other side of the lever, and move it to or from the point of support until the lever balances about that point. (4) Note the magnitude of the weights, and their distances from the point of support, also the length of the lever. (5) Draw a structure diagram (Fig. 28*a*), and draw the lines of action of the weights. (6) Draw a force diagram (Fig. 28*b*) for the weights, and a polar diagram as before. (7) Draw a funicular polygon over the lines of action of the forces. (8) Produce the first line of the funicular polygon OA, and the last line DO to meet in the point r. (9) Then r is a point through which the resultant must pass; and as we know its direction is vertical, we can draw its line of action through r. (Note that as the lever is balanced at its centre, the point r must be on the line of action of the weight of the lever). (10) Measure the distance of the resultant line from the line of action of the weights, and compare with the distance as measured from the experiment. (11) The magnitude of the resultant is evidently represented by the line da, the closing line of the force polygon.

RECORD OF EXPERIMENT. *To find the resultant of two parallel forces of 4 lbs. and 7 lbs. acting on a beam.*

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As found by drawing, the resultant passes through the point r, and is 12″ from the weight of 7 lbs., and 21″ from the weight of 4 lbs. By experiment the distances are 12″ and 21″ respectively, and the resultant passes through the balancing point of the lever.

Weight of lever, 1 lb. 6½ ozs. Resultant = 12 lbs. 6½ ozs.

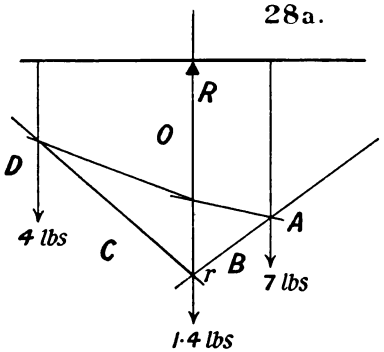

FIG. 28a.

STRUCTURE DIAGRAM.
Scale, $\frac{1}{10}$.

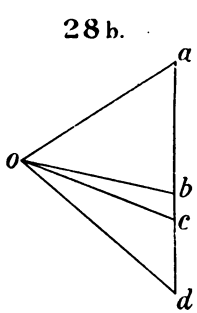

FIG. 28b.

FORCE DIAGRAM.
Scale, 1 cm. = 2 lbs.

EXERCISES.

The following exercises are to be worked graphically :—

12. A uniform horizontal bar, 12 ft. long, weighs 20 lbs., and is supported at each end, while 3 ft. from one end a weight of 50 lbs. is hung. Find the reactions between the bar and its supports.

13. Vertical forces 1, 2, 3, 4, 5, and 6 lbs. act at the angular points of a regular hexagon. Find their resultant.

14. Forces of 4, 8, 6, and 10 lbs. act vertically downwards at equal distances along a horizontal line. What is the point of application of their resultant?

15. A uniform plank is placed across two trestles, the middle of the plank being midway between the trestles. Find how the pressure on a trestle changes as a man whose weight is five times that of the plank walks along the plank from one trestle to another.

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16. A beam $ABCDE$ has a vertical supporting force at A. At E there is a pin-joint support. AB is 5′, BC is 2′, CD is 6′, DE is 4′. There are vertical loads of 2 tons at B, and 3 tons at D, and at C there is a load of 5 tons inclined at 30° to the vertical, its horizontal component being towards A. All forces in one plane. Find the supporting force, graphically.

17. A uniform iron rail, which weighs 100 lbs., and is 15 ft. long, is supported in a horizontal position on two posts, which are 5 ft. and 4 ft. respectively from the ends of the rail. Find the pressures on the posts (a) when a weight of 50 lbs. is hung from one end of the rail, (b) when it is taken off this end and hung from the other end.

18. The fly-wheel of an engine weighs $2\frac{3}{4}$ tons: it is carried by a shaft supported on two bearings, each 9 inches long and 2 feet 6 inches apart from centre to centre. Assuming that the centre of the boss of the wheel is 13 inches from the centre of the nearest bearing, find how much of the load each bearing carries. It is assumed that the shaft is sufficiently stiff for the pressure to be uniformly distributed over the whole length of each bearing.

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CHAPTER IV.

MOMENTS. LEVERS.

EXPERIMENT I. **To find what is meant by the "moment of a force."**

APPARATUS. (Fig. 29.) A beam of hard wood 5′ long, $1\frac{1}{2}''$ deep, and $\frac{3}{4}''$ thick, with the upper edge divided into lengths of 1″, a slight saw-cut being made at every inch to form a nick. A piece of $\frac{1}{2}''$ square steel bar is driven through the centre of the beam, the ends being drilled slightly hollow and conical, to form centres, about which the beam will turn as an axis. A wood bracket from the wall carries a plate-iron bracket, in which are screwed two set screws with hardened conical ends, to fit into the centres of the axis of the beam. Two pieces of iron bar fixed on one side of the bracket act as stops to keep the beam in place.

FIG. 29.

Method. (1) Balance the beam on its centres, tying on a small weight on one side, if necessary, to produce a balance. (2) Sling a 4 lb weight on one side of the beam at a distance of 12″ from the axis. (3)

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Take a 2 lb. weight and sling it on the other side of the beam, and move it to or from the axis until the beam is again in balance. Measure and write down the distance of the 2 lb. weight from the axis. If the balancing is carefully done, this distance ought to be 24″; that is, a weight of 4 lbs. on one side at a distance of 12″ was balanced by a weight of 2 lbs. on the other side at a distance of 24″; or if half the weight is used, it must act at double the distance to produce a balance. (4) Remove the 2 lbs. weight, and perform exactly the same experiment with a 4 lb. weight. The distance from the axis of this 4 lbs. weight ought to be 12″; that is, if the same weight is used, it will produce balance if at the same distance. (5) Remove the 4 lbs. weight, and repeat the experiment with a weight of 8 lbs. The distance of the 8 lbs. weight from the axis ought to be 6″; that is, a weight of 4 lbs. on one side at a distance of 12″ was balanced by a weight of 8 lbs. on the other side at a distance of 6″; or if double the weight is used, it need only act at half the distance.

What this Experiment teaches. Now whether the student obtains results quite as accurate as these or not, they certainly ought to be near enough to suggest to him that the effect of a force when turning a body about a point or axis depends not only upon the magnitude of the force, but upon its distance from the point about which turning takes place. The results show clearly that the greater the distance the less need the force be, and the less the distance the greater must the force be. Evidently, then, when a force is tending to turn a body about some point or axis, we must consider two things, (*a*) the magnitude of the force, (*b*) its distance from the point or axis; and in expressing the measure of the turning effect we must multiply these two things together. The quantity thus produced is called the "*moment of the force.*" Thus in the experiment just performed the moment of the weight of 4 lbs. at a distance of 12″ is $4 \times 12 = 48$ units, the moment of the weight of 2 lbs. at a distance of 24″ is $2 \times 24 = 48$ units, and the moment of the weight of 8 lbs. at a distance of 6″ is $8 \times 6 = 48$ units.

DEFINITION. *The* **moment** *of a force about a point is the product of the force into the perpendicular distance of its line of action from the point.*

Units of Moments. If the unit of force is the weight of one pound, and the unit of distance is one foot, then the unit moment is one pound × one foot = one pound-foot (1 lb.-ft.). It is better to use lb.-foot than foot lb., as the latter is a unit of work.

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Clockwise and Anticlockwise Moments. By reference to the conditions of the experiment just performed, the student will see that the weight on the left-hand side of the beam is tending to turn the beam round in a direction opposite to the hands of a clock; while the weight on the right-hand side is tending to turn the beam in the same direction as the hands of a clock. The two moments evidently act against each other, and for this reason they are often called positive (+) and negative (−) moments. It is immaterial which is which, but the following definitions are general:—

Forces which tend to produce turning in the **same** *direction as the hands of a clock are said to have* **clockwise** *or* **negative** *moments. Forces which tend to produce turning in the* **opposite** *direction to the hands of a clock are said to have* **anticlockwise** *or* **positive** *moments.*

EXPERIMENT II. **To find the conditions of equilibrium of a number of forces acting on a body which can turn about a fixed point or axis.**

APPARATUS. (Fig. 30.) An irregular-shaped piece of thin card or board about 10″ × 8″, with holes pierced in four

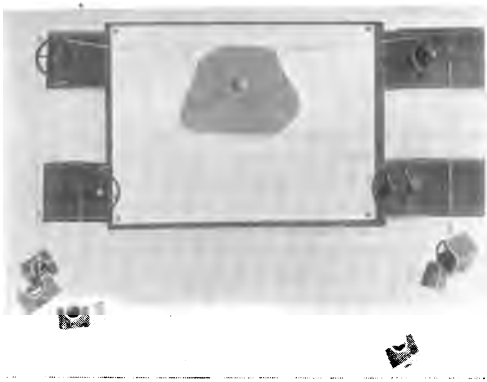

FIG. 30.

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positions near the edge. Pieces of silk cord are tied to the board through these holes, and are placed over the pulleys of the Polygon of Forces board, Fig. 18, with weights tied to their other ends.

Method. (1) Pin a piece of drawing-paper on the pulley board. (2) Fix the piece of card or board representing the body to be acted upon about the centre of the pulley board by means of a fine bradawl, or a stout long drawing or drugget pin. Make the hole in the card large enough to allow the card to turn quite freely in a vertical plane. (3) Hang the cords over the pulleys, and tie on any convenient weights, say of 2, 3, $2\frac{3}{4}$, 2 lbs., so that the card assumes a position of equilibrium somewhat as shown in Fig. 30. (4) Mark the drawing-paper under each cord to give the direction of the cords, and note the weight acting on each. (5) Remove the paper, and draw lines representing the direction of the cords, which will, of course, be the lines of action of the forces acting on the card. (6) From the line of action of each force draw a perpendicular line to the small hole in the paper corresponding to the axis at which the card was fixed. (7) Measure the length of these perpendiculars, and calculate the moment of each of the four forces, adding together like moments. (8) Compare the sum of the clockwise moments with the sum of the anticlockwise moments, and write down the result.

RECORD OF EXPERIMENT. *Experiment on the conditions of equilibrium of four forces of* 2, 2, $2\frac{3}{4}$, *and* 3 *lbs. when acting on a body which can turn freely round a fixed axis:*

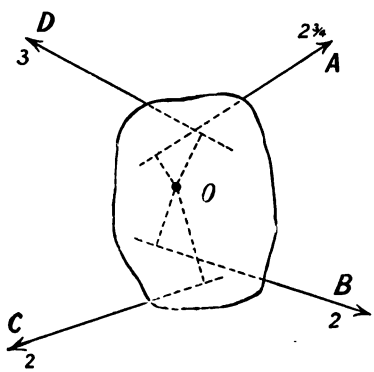

FIG. 31.

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Moments about the Turning-Point O.

Moments of A and C are clockwise, or −
„ B and D are anticlockwise, or +

Moment of	$A = 2{\cdot}75$ lbs.	$\times 0{\cdot}38'' =$	1·045 lb. in.
„	$C = 2$	$\times 3{\cdot}12 =$	6·24 „
Total clockwise moments			= 7·285 „
Moment of	$B = 2$ lbs.	$\times 1{\cdot}8'' =$	3·6 „
„	$D = 3$	$\times 1{\cdot}20 =$	3·6 „
Total anticlockwise moments			= 7.2

Result. The sum of the clockwise moments is equal to the sum of the anticlockwise moments, and therefore the algebraic sum is zero.

What this Experiment teaches. After working this experiment as described, the student should have no difficulty in fully understanding and appreciating the following propositions relating to moments, which should be directly suggested by the results of the experiment, and of which the experiment itself may be regarded as a verification. The mathematical proof of such propositions is, of course, a part of Theoretical Mechanics :—

I. *When a body which can turn about a fixed point is in equilibrium under the action of any number of forces in the same plane, the algebraical sum of the moments of the forces about that point is zero; that is, the clockwise or − moments are together equal to all the anticlockwise or + moments.*

II. *When a body is in equilibrium under the action of any number of forces in the same plane, then the algebraical sum of the moments of the forces about any point in that plane is zero; that is, the sum of the moments of the forces which tend to turn the body in one direction about any point is equal to the sum of the moments of the forces which tend to turn the body in the opposite direction about the same point.*

III. *The algebraic sum of the moments of any number of forces acting in the same plane about any point in the plane is*

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equal to the moment of the resultant of these forces about the same point.

(Note.—The student can verify this last statement by reference to the conditions of Experiment II. Any one of the forces is the equilibrant of the other three; and if its direction is reversed, it is the resultant.)

The second of these propositions is known as the *Principle of Moments.*

EXPERIMENT III. **To find by experiment and by the principle of moments the reactions at the supports of a beam when acted upon by two parallel forces.**

APPARATUS. The graduated wood beam, with its ends supported by circular spring balances, as used for Experiment IV., Chapter III., Fig. 27.

Method. (1) Take weights of 14 and 28 lbs to represent the forces. (2) Read the balances so that the weight of the beam may be neglected. (3) Tie the 14 lb. weight to a point 12″ from one end, and the 28 lb. weight to a point 20″ from the other end. (4) Adjust, if necessary, until the beam is horizontal. (5) Read the balances, and write down the reactions at each end. (6) Calculate the moments of all the forces about each end of the beam separately by calling the reactions at the ends P and Q units, and then finding their numerical value; thus:—

Length of lever 58″ End P nearest 14 lbs. weight and 12″ away. End Q nearest 28 lbs. weight and 20″ away.

Moments about Q.

$$P \times 58'' = (14 \times 46) + (28 \times 20)$$

$$\therefore\ P = 20{\cdot}76 \text{ lbs.}$$

Moments about P.

$$Q \times 58 = (28 \times 38) + (14 \times 12).$$

$$\therefore\ Q = 21{\cdot}24 \text{ lbs.}$$

Note that it is only necessary to take moments about one end; thus if P be found, then $Q = (7 + 14) - P$. Compare the reactions as found in this way with the reactions as found by experiment.

NOTE.—If it is true that the positive and negative moments about any point in the plane are equal, then the moments of the reactions P and Q and of the 14 lb. weight about the point of suspension of the 7 lb. weight must be equal; and also the moments of the reactions and of the 7 lb. weight about the point of suspension of the 14 lb. weight. The student should test this with the above, and also with Experiment I, by taking moments about a point midway between the axis and the 2 lb. weight, noting that the supports are evidently pushing upwards with a force of $4 + 2 = 6$ lbs., omitting weight of lever.

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EXERCISES.

1. Draw a square, whose angular points in order are A, B, C, D, and whose side is 12 units long. A force of 7 units acts from A to B, and a force of 10 units acts from A to D. (a) If the moments of the forces are taken about C, will they have the same signs, or opposite signs, and why? (b) Find the algebraical sum of the moments about C. (c) Find the point in CD about which the moments are equal, but of opposite signs.

2. A, B, C, D are the angular points taken in order of a square. Produce AD to E, making AE equal to $2AD$. Suppose a force of 10 units acts from B to A, and a force of 20 units from B to C. Find the moment of their resultant about D, also about A, and about E.
Along what line does the resultant act?

3. A, B, C, D is a rectangular body, which can move freely in a vertical plane round a hinge at A. AB is 10 ft. long, and BC is 6 ft. The body weighs 300 lbs.; it rests with AB against a fixed point P vertically below A, and at a distance of 8 ft. from A, find (1) the moment of the weight with respect to A, (2) the reaction of P against the body.

4. What is meant by the moment of a force with respect to a point? If there are two moments, and one is marked with a positive sign and the other with a negative sign, what do these signs show?
$ABCD$ is a square, and AC a diagonal. In AC take a point O, such that AO is a third of AC. Forces of 20, 7, and 5 units act from A to B, C to B, and D to C respectively. If the side of the square is 6 units long, write down the moment about O of each force with its proper sign, and find their sum.

5. Draw a square $ABCD$, and a diagonal BD. Forces of 10, 12, and 20 units act from B to C, D to C, and B to D respectively. If the side of the square is 3 units long, find the moment with respect to A of the resultant of the three forces, and state in what direction the forces tend to turn the square round A.
Define the moment of a force with respect to a point.

6. A, B, C, D are the angular points of a square taken in order. A force of 20 units acts from A to B, and a force of 25 units from A to D. Find the moment of the resultant about the middle point of DC, a side of the square being 6 units long.

7. Define the moment of a force with respect to a given point. Draw an equilateral triangle ABC, and let a side represent a length of 3 ft.; a force equal to the weight of 10 lbs. acts along BC. Find the numerical value of its moment with respect to A. The force may act either from B to C, or from C to B. How would the moment in the one case be distinguished from the moment in the other case?

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LEVERS.

Levers are usually regarded in Mechanics as machines, since they are generally used in practical life to do work, such as in the common example of a crowbar when used for moving heavy weights.

Force and Load in a Lever. Under ordinary practical conditions there are two external forces acting on the lever, one due to the resistance overcome, usually called the *Weight*, and the other to the force or effort which moves the lever, usually called the *Power*. But the use of these terms, especially of the latter, is very confusing, since the real use of the word "power" is to express the rate of doing work. It is better to call the two forces "*Load*" and "*Force*." There is also the reaction of the point about which the lever turns.

Fulcrum. The fixed point in the lever about which it turns is called the *fulcrum*.

Arms of Lever. The perpendicular distances of the line of action of the "load" and "force" from the fulcrum are called the *arms* of the lever.

Principle of Lever. Assuming the two forces acting on the lever to be the "load" and "force," this principle is usually stated as follows: "*When a lever is in equilibrium, the force multiplied by its distance from the fulcrum is equal to the load multiplied by its distance from the fulcrum.*" This follows directly from the principle of moments.

Experiments on Levers.

Apparatus. A simple form of straight lever, as described in connection with Experiment I., Fig. 29. The fulcrum is a separate piece, and can be fixed in any position on the lever by set screws, which grip into the upper and lower edge of the lever. The weights are best hung on by loops of thin string.

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Bell Crank Lever. A simple form of this for experimental use is shown in Fig. 32. The wood lever is centred on simple coned points, the long arm measuring 30″, and the short arm to the point to which the spring balance is fixed, measuring 6″. The length of the balance is adjusted by the tightening screw and wing nut shown. The base board is about 30″ × 9″.

FIG. 32.

The following will illustrate the method of working an experiment on levers, and recording the result.

EXPERIMENT IV. **A lever with the fulcrum at the centre is loaded with a 7 lb. weight at a distance of 14″ from the fulcrum. To find the force required to produce balance at a distance of 20″ from the fulcrum.**

Method. (1) See that the lever balances when unloaded, and if not, balance it with a small weight. (2) Suspend the 7 lb. weight at the given distance on one side of the fulcrum, and suspend a scale-pan from the given distance on the other side. (3) Add weights to the scale-pan until the lever is balanced. (4) Work out and enter the results as follows :--

RECORD OF EXPERIMENT. *Experiment with a lever to find force required to balance a load of 7 lbs. at a distance of* 14″ *from the fulcrum when the distance of the force from the fulcrum is* 20″.

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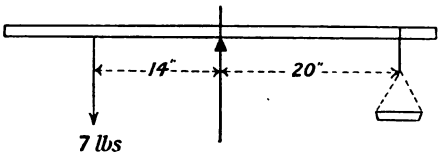

FIG. 32a.

Weight of scale-pan = 0·69 lbs.
Weights in „ = 4·19
Total force = 4·88

Load.	Arm of Load.	Moment of Load.	Force.	Arm of Force.	Moment of Force.	Difference of Moments.
7 lbs.	14	98 units	4·88 lbs.	20″	97·6 units	0·4 units

EXPERIMENT V. **Work the following experiments with a lever with fulcrum at centre. In each case show that the principle of moments holds true.**

(*a*) Given load = 7 lbs.; arm of load = 12″; force = 4 lbs. Find arm of force.

(*b*) Given load = 11 lbs.; arm of load = 10″; arm of force = 25″. Find force.

(*c*) Given a load of 7 lbs. at 10″ from fulcrum, and a load of 4 lbs. at 20″ from fulcrum. Find magnitude of force when its arm is 24″.

EXPERIMENT VI. **A lever fixed at one end is acted upon by a force of 7 lbs. acting downwards at a distance of 20″ from the fulcrum. Find the magnitude of the force required to balance it at a distance of 40″ from the fulcrum, and show that the moments of opposite sign are equal.**

NOTE.—The weight of the lever must be considered, and may be regarded as acting at the centre of the lever. The upward force required must be applied by a spring balance, suspended above the lever.

EXPERIMENT VII. **Work the following experiments with the bell crank lever of Fig. 32, and show that the principle of moments holds in each case :—**

(*a*) Given a load of 4 lbs. at a distance of 20″ from the

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fulcrum. Find the force at the end of the short arm required to produce balance

(*b*) Find the force required at a distance of 16″ from the fulcrum to balance a force of 12 lbs. at the end of the short arm.

EXERCISES ON LEVERS.

8. A uniform beam, 24 ft. long and weighing 200 lbs., is supported on two props, one 6 ft. from one end, the other 9 ft. from the other end of the beam. Calculate the pressure on each prop when a man weighing 180 lbs. stands as near this latter end as he can without upsetting the beam.

9. A uniform beam, 12 ft. long and weighing 56 lbs., rests on and is fastened to two props 5 ft. apart, one of which is 3 ft. from one end of the beam. A load of 35 lbs. is placed (*a*) on the middle of the beam, (*b*) at the end nearest a prop, (*c*) at the end furthest from a prop. Calculate the weight each prop has to bear in each case.

10. A uniform beam, 14 ft. long and weighing 120 lbs., is attached to two props, one of which is 3 ft., and the other 5 ft. from its centre. Calculate the forces on the props when a weight of 100 lbs. is placed first at one end, and then at the other end of the beam.

11. A bar projects 6 inches beyond the edge of a table, and when 2ozs. is hung on to the projecting end, the bar just topples over. When it is pushed out so as to project 8 inches beyond the edge, 1 oz. makes it topple over. Find the weight of the bar, and the distance of its centre of gravity from the end.

12. The length of a weightless lever is 12 inches, and masses m and $2m$ are attached to its ends. Find where the fulcrum must be placed in order to effect a balance.

13. What force must be applied to the longer arm of a lever whose arms are as 5 : 1 in order to balance a weight of 12 lbs suspended from the shorter arm?

14. A uniform lever weighs 6 lbs., and from one end a weight of 12 lbs. is suspended. The length of the lever being 18 inches, find the point in its length about which the whole system will balance.

15. A rod 16 ft. long balances about a point 4 ft. from one end; and if a weight of 120 lbs. be attached to the other end, it balances about a point 6 ft. from that end. Find the weight of the lever.

16. A ladder is supported by two men, one of whom is at one end of the ladder. Where must the other be placed that he may bear two-thirds of the weight?

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17. To a uniform rod weighing 3 lbs. weights of 6, 8, 10, and 12 lbs. are attached at equal distances, the weight of 8 lbs being at the middle point. Find the point about which the rod will balance.

18. A rod 18 inches long can turn about one of its ends, and a weight of 5 lbs. is fixed to a point 6 inches from the fixed end. Find the force which must be applied at the other end to preserve equilibrium.

19. A weightless rod AB rests horizontally on two points under A and B 14 ft. apart. It carries a weight suspended from a point X, which causes a pressure of 3 units on A, and of $7\frac{1}{2}$ units on B. Find the distance of X from A.

20. State the principle of the lever. A uniform straight bar, 14 inches long, weighs 4 lbs. It is used as a lever, and an 8 lb. weight is suspended at one end. Find the position of the fulcrum when there is equilibrium.

21. Draw an equilateral triangle ABC, and let BC represent a weightless lever acted on at B by a force of 7 units from A to B, and at C by a force of 9 units from A to C. If the lever is at rest, find by construction or otherwise the position of the fulcrum. Find also the magnitude and direction of the pressure on the fulcrum.

22. The diameter of the safety-valve of a steam boiler is 3 inches. The weight on the end of the lever is 55 lbs., and the distance from the centre of the valve to the fulcrum is 4·5 inches. What must be the length of the lever from the centre of the valve to the point of suspension of the weight in order that the valve will just lift when the pressure of steam in the boiler is 80 lbs. per square inch? Neglect the weight of the lever and the valve.

23. Draw a horizontal line AB, and a line BC making an angle of 30° with it, C being above A. Suppose that AB represents a lever without weight that can turn freely round a hinge at A. A weight of 15 lbs. is hung from a point D on the lever, AD being three-quarters of AB. Find by construction or otherwise the force acting along BC which will support the weight. Specify completely the force when you have determined it.

24. State the principle of the lever, and prove it when P and W act on opposite sides of the fulcrum. A weight of 5 lbs. is hung at one end of a uniform bar, which is balanced over a knife-edge at a point 14 inches from the end at which the weight hangs. The bar weighs 30 lbs. Find its length.

25. Prove the principle of the lever. A uniform beam 10 ft. long weighs 500 lbs., and is supported at its extremities. Find the bending moment tending to break it at a point 4 ft. from one end.

26. Define "moment of force." How is it measured? A bar of metal of uniform section weighs 5 lbs., and a weight of 10 lbs. hangs from one end. It is found that the bar balances on a knife-edge at 9 inches from the end at which the weight hangs. What is the length of the bar?

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27. Let AB be a horizontal line 10 inches long, and F a point in it 6 inches from A. Suppose that AB is a lever that turns on a fulcrum under F, and carries a weight of 50 lbs. at B. If it is kept horizontal by a fixed point above the rod 5 inches from F, and 1 inch from A, find the pressure on the fulcrum, and on the fixed point.

28. A uniform straight lever, 4 ft. long, weighs 10 lbs.; the fulcrum is at one end. Find what upward force acting at the other end will keep the lever horizontal when a weight of 10 lbs. is hung at a distance of 1 ft. from the fulcrum. Find also the pressure on the fulcrum, and the direction in which it acts.

29. The handle of a whip is 56 inches long, and weighs 12 ozs. It will balance on a point 21 inches from one end; it is supported horizontally on two points, one under each end. Find its pressure on each point of support.

30. A rod or lever is capable of turning freely round a fixed point or fulcrum, and is acted on by a force at each end. Putting the weight of the lever out of the question, state the relation which must exist between the forces when the rod stays at rest.

31. Find at what point of its length a heavy uniform rod 18 ft. long, and weighing 5 lbs. per foot, will balance when 150 lbs. is placed at one end, and 60 lbs. at the middle.

32. A beam 24 ft. long balances itself about a point one-third of its length from the thicker end; but when a weight of 6 lbs. is suspended from the thinner end, the prop must be moved 3 ft. towards this end, in order to maintain equilibrium. Find the weight of the beam.

33. In a lever safety-valve, if the lever be 16 inches in length, and the centre of the valve-seat is 4 inches from the fulcrum, while the diameter of the valve is 4 inches, find the weight to be placed at the end of the lever so that steam may blow off at a pressure of 45 lbs. per square inch, the weight of the valve and of the lever being neglected.

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CHAPTER V.

THE USE OF SQUARED PAPER. CURVES TO EQUATIONS. EQUATIONS TO CURVES. SPACE VELOCITY AND ACCELERATION CURVES.

Graphical Representation of Results of Experiments. Squared paper allows of the graphical representation of experimental results with great ease. The object of many experiments is to discover the relation existing between two things whose value depends on one another; we want to know the law of dependence. For example, we may require to know how the deflection of a beam, or the extension of a wire depends upon the load acting upon it; or how the friction between two surfaces depends upon the pressure between them; or how the brake horse-power of an engine varies with its indicated horse-power; or we may require to show the relation between metric and English units of length; or how the height of the barometer and the temperature varies at different hours of the day, an example of which can be seen in any daily paper. These and numerous other results can all be represented on squared paper in a manner which will show a great deal more than their mere arrangement in columns of figures.

Squared paper is so called because it is divided by horizontal and vertical lines into small squares, which for the purpose we are concerned with are usually of $\frac{1}{10}''$ side. Every tenth line is more strongly marked than the others, thus dividing the paper into square inches, and assisting in counting. Certain makes of paper have every fifth line also distinguished by stronger marking. For convenience of reproduction the squared paper used for the diagrams in this chapter was divided into squares of $\frac{1}{5}''$ side.

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Example 1. An experiment on the force required to raise different loads with a chain pulley block gave the following results: —

Load (lbs.)	56	112	168	224	280	336	392	448	504
Force (,,)	$12\frac{1}{4}$	$20\frac{1}{2}$	$29\frac{1}{2}$	$37\frac{1}{2}$	46	$54\frac{1}{2}$	63	$71\frac{3}{4}$	80

Plot the results on squared paper to show the relation between the force and the load.

Method. We have first to decide the scale to be used for representing the load and force, and looking at their least and greatest value we see that a scale of 1″ to 56 lbs. for the load will require a length of 9″ to represent the greatest load, and a scale of 1″ to 20 lbs. for the force will require a length of 4″ to show the greatest force. Choose then a piece of squared paper about 10″ by 6″, and mark a point *O* (Fig. 33) near the bottom left-hand corner at one of the intersections of the dark horizontal and vertical lines which mark the inch divisions. From this point mark every inch length to the right along the dark horizontal line with the figures 56, 112, 168, etc., showing the different values of the load. Now mark a point on each vertical line passing through the "load points" to represent the force corresponding to that load to the scale taken. Thus the force for the load of 112 lbs. is $20\frac{1}{2}$ lbs., and if 1″ represents 20 lbs., this will be represented by a length of $\frac{20{\cdot}5}{20} = 1{\cdot}025$″. Therefore, mark a point on the vertical line, passing through the load point marked 112, at a distance of 1·025″ from that point. This can be done by moving up to the next dark line, which will show 1″, and then estimating a distance of $\frac{1}{4}$ the side of a small square above this inch line to show 0·025″. It is best to mark the points representing the force by a small × or o, and after a little practice the student will readily be able to mark off fractions of one of the small squares. Proceed in this way for each of the nine different forces. Now a glance at the points plotted will show that they appear to lie all in the same straight line, and we are safe in assuming that but for errors in the experiment, or in plotting the points, they would lie all in one straight line. We therefore draw the straight line which best passes through the points. In doing this, the line will probably pass through some of the points, and it will have some above and some below it; those above the line should be as much above as those below are below. A very easy way of drawing this line is by using one of the transparent celluloid set squares, as points covered by the set square are still visible.

In Fig. 33 the line has been drawn passing through four of the force points, and the student should endeavour to realize how much more effectively it shows the relation between the force and the load than do the lines of figures given in the question.

In addition to showing how the force varies with the load, the line also allows us to find the force corresponding to any given load. For example, the force corresponding to the load represented by the distance *OX*, is represented by the distance *XY*, and so on for any other load.

The line of Fig. 33 is a straight line, but for many experiments and plotting of results it may be a regular or irregular curve. Whatever the

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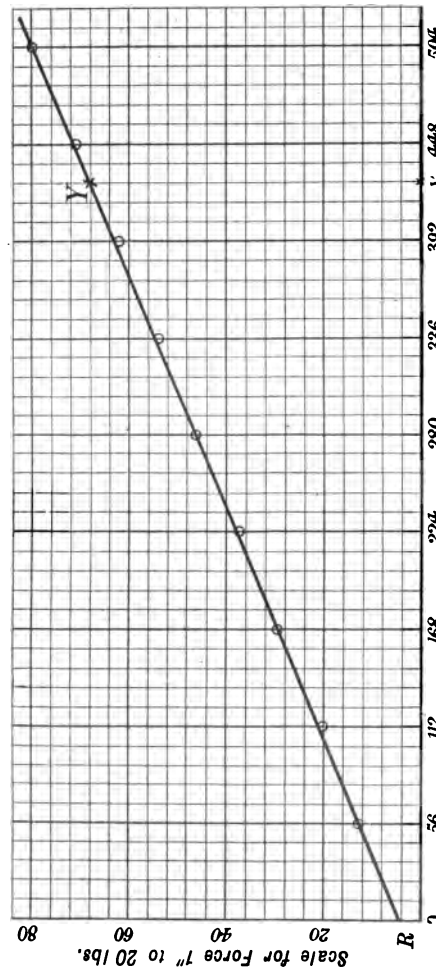

FIG. 33.

Curve showing relation between Force and Load in a chain pulley block.

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form of the lines, they are generally termed "curves," and the method is termed "plotting or drawing curves."

The point *O* (Fig. 33), from which the load points were started, is called the *origin* or *zero point* of the curve.

The line along which the load points are marked is called the *base line*, or *abscissa* of the curve, and the vertical lines on which the force points are marked are called *ordinates*.

NOTE.—The paper should always show what the curve is, and the scales used, as in Fig. 33.

Example 2. The tensile test of a bar of iron after reaching the elastic limit gave the following results :—

Load (tons)	15	16	17	18	19	20	21
Extension (inches)	0·15	0 18	0·21	0·28	0·31	0·39	0·44
Load (tons)	22	23	24	25	26	27	27·6
Extension (inches)	0·53	0·61	0·71	0·89	1·10	1·5	2·31

Draw a curve on squared paper showing how the extension varies with the load.

NOTE.—Use a scale of 1″ to 1 ton load and 1″ to $\frac{1}{4}$″ extension. After plotting the points for the extension, draw the continuous smooth curve which best passes through all the points as shown in Fig. 34.

Example 3. An experiment on the resistance of the friction of a silk tape round a wooden peg, with different loads, gave the following results :—

Load	(grammes)	19·6	39·6	59·6	79·6	99·6
Resistance of Friction	,,	29	46	63	78	95
Load	,,	119·6	139·6	159·6	179·6	199·6
Resistance of Friction	,,	105	123	130	140	150

Plot these results on squared paper, showing how the friction varies with the load.

NOTE.—The fractional values for the load are due to the use of a scale-pan which weighed 9·6 grammes, and which formed part of the load. If the load is marked off from an origin, as in Examples 1 and 2, the points will not correspond to any vertical line. This can be avoided by marking a distance to represent 9·6 grammes to the left of the point *O* (Fig. 33), and then marking points to the right of *O*, to represent 10, 30, 50 grammes, etc.; the point which is 9·6 grammes to the left of *O* will then be the origin or zero point.

Example 4. The Meteorological Office observations for barometric pressure and for temperature for January 6th and 7th, 1898 were as follows :—

	p.m.	a.m.	a.m.	a.m.	noon.
Time, Jan. 6th	12	3	6	9	12
Height of barometer″	29 9	29·9	29·88	29·94	29 95
Shade temperature F°	51	52·8	52	52·8	55

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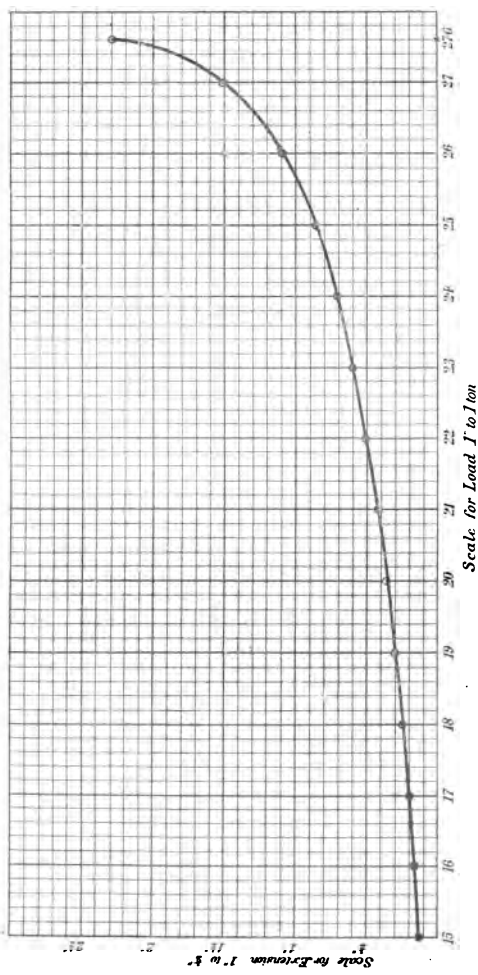

FIG. 84.
Curve showing extension of iron bar.

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	p.m.	p.m.	p.m.	p.m.
Time	3	6	9	12
Height of barometer	29·95	29·94	29·93	29·9
Shade temperature	53·9	53·5	52	50
	a.m.	a.m.	a.m.	noon.
Time, Jan. 7th	3	6	9	12
Height of barometer	29·84	29·87	30·01	30·06
Shade temperature	50·2	49	48	47·2
	p.m.	p.m.	p.m.	p.m.
Time	3	6	9	12
Height of barometer	30·12	30·18	30·2	30·21
Shade temperature	49·2	49	49	49·1

Plot curves showing the variation in the temperature and height of the barometer for the period stated.

NOTE.—The base line may represent a barometer height of 29·5″, and a temperature of 40°. Take a scale of at least 1″ for 2° of temperature, and for $\frac{1}{10}$″ height of barometer.

Drawing Curves to Equations. Certain equations can be represented by curves in such a way that the curves show the root or roots of the equations, so that the equations may be solved by drawing curves to them.

For example, take the simple equation

$$3x + 15 = x + 25$$

In order to solve this in the usual way we should express it in the form $2x = 10$, from which we see at once that $x = 5$. But if $2x = 10$, then also $2x - 10 = 0$. If we state this as $2x - 10 = y$, we have only to find the value of x when $y = 0$, and the equation is solved. This we can do by assuming different positive or negative values for x, until we find such a value for it that $y = 0$.

Thus if $x = +1$, $y = 2 - 10 = -8$
$= +2$, $y = 4 - 10 = -6$
$= +3$, $y = 6 - 10 = -4$
$= +4$, $y = 8 - 10 = -2$
$= +5$, $y = 10 - 10 = 0$
$= +6$, $y = 12 - 10 = 2$
$= +7$, $y = 14 - 10 = 4$

Now having obtained seven different values for x and y, let us plot these on squared paper, exactly as for curves already

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drawn, taking suitable scales, say of $\frac{1}{2}''$ to 1 for x along the abscissa or base line, and $\frac{1}{2}''$ to 1 for y along the ordinates. This curve is shown in Fig. 35 *A*, negative values of y being taken below the base line, and positive values above the line. Therefore at the point *5*, where the curve crosses the line $y = 0$, and the length 05 from the origin 0 represents 5, or the value of x, which satisfies the given equation.

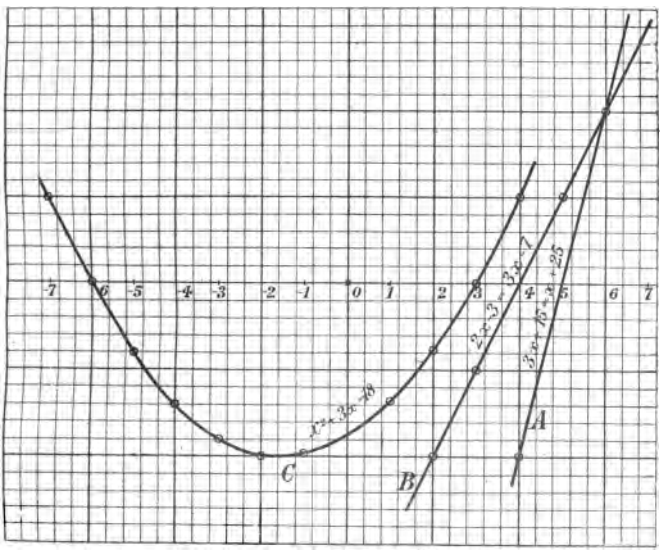

FIG. 35.

Curves to Equations.

Fig. 35 *B* is a curve to the equation $2x - 3 = 3x - 7$, and is plotted by finding the value of y for different values of x, exactly as already described.

Fig. 35 *C* is a curve drawn to the quadratic equation $x^2 + 3x = 18$. This can be expressed as $x^2 + 3x - 18 = y$, and any values for x which make $y = 0$ will satisfy the equation. Calculate as before the values of y when x is $+1$, $+2$, $+3$,

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etc. If y becomes larger and larger with these increasing positive values of x, then find y for different negative values of x. Then plot the curve as before, using suitable scales, noting that negative values of x are to be plotted to the left of the origin. Plot enough points so that the curve will cut the line twice, then the number represented by the length $O3$ is one root of the equation, and the length of $O6$ is the other root. The former is $+3$, and the latter is -6.

Simple Simultaneous Equations. In order to draw curves to simple simultaneous equations, each equation should be treated as a simple equation, and a curve drawn to it, exactly as already described. The intersection of the two curves being common to both, show the value of x and y, which solve both equations.

Examples on Drawing Curves to Equations. Draw curves to the following equations, and find the roots of the equations from the curves:—

1. (*a*) $3x+4=5(x-2)$. (*b*) $-\frac{x}{3}=2$.

(*c*) $\frac{x}{18}=\frac{9}{42}$. (*d*) $\begin{matrix} x+2y=13. \\ 3x+\ y=14. \end{matrix}$

(*e*) $\begin{matrix} 6y-5x=18. \\ 12x-9y=0. \end{matrix}$ (*f*) $\begin{matrix} 2x+\ y=0. \\ \frac{1}{2}y-3x=8. \end{matrix}$

(*g*) $x^2=10x-21$. (*h*) $x^2-\frac{2}{3}x=32$.

(*i*) $9x^2-143-6x=0$.

2. Draw curves to the following equations, finding at least 10 points for each curve: (*a*) $v=gt$; (*b*) $s=\frac{1}{2}gt^2$; (*c*) $v^2=2gs$, when $g=32$ in each case. (Note.—In (*a*) and (*b*) find the value of v and s when t increases from 1 to 10. Set off t along the base line, and the values of v and s along the ordinates. (*c*) Set off s along the base line, and the value of v^2 along the ordinates).

3. Draw a curve to the equation $P=1{\cdot}4+{\cdot}3W$, for values of W of from 0 to 10. Set off W along the base line, and P along the ordinates.

Finding Equations to Simple Curves. The curves, as given in Figs. 33, 34, are simple curves with ordinates at right angles (rectangular ordinates). Since the curves represent graphically the relation between two quantities which vary with one another, the equation to the curve will represent the

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algebraical relation between them. It is necessary to know this equation in order that the value of one of the quantities may be found when the other is known.

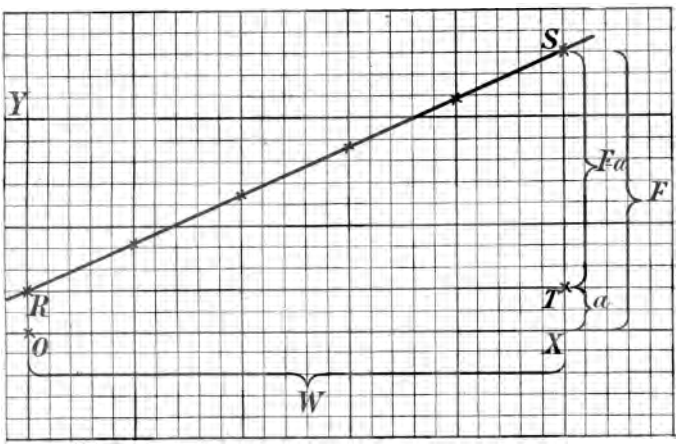

FIG. 36.

Let the figure represent a curve drawn to represent the relation between the load W and the friction F of a pulley as found by experiment. W is plotted along OX, and F along OY, the curve being the best straight line that can be drawn through the points representing the different values of F.

Draw a line RT through R, parallel to the base line OX, and let $OR = a$, $\therefore XT = a$;

$$\text{also } OX = W,\ SX = F;\ \text{and } ST = F - a$$

$$\frac{ST}{RT} = \frac{ST}{OX} = m \text{ (a constant)}$$

$$\therefore \frac{F-a}{W} = m, \text{ that is, } \underline{\underline{F = m\ W + a}}.$$

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By substituting numerical values for F, a and W, the values of m can be found, and thus the equation contains two unknowns, F and W; and the value of F can be found for any given value of W.

After finding the equation, it should be tested by substituting a value of W corresponding to a point on the curve, and seeing if the found value of F agrees with its known value.

Exercise. To find the equation to the curve of Fig. 33, that is, to find the equation expressing the relation between the force and load.

Let W = load, and P = force.

Take the point in the curve, where $W = 448$ lbs., and $P = 71{\cdot}75$ lbs. Then a is represented by the distance OR, which measures $0{\cdot}175''$, and therefore represents $3{\cdot}5$ lbs., since $1'' = 20$ lbs.

$$\therefore m = \frac{P - a}{W} = \frac{71{\cdot}75 - 3{\cdot}5}{448} = {\cdot}152;$$

$$\therefore P = 0{\cdot}152\ W + 3{\cdot}5.$$

If this equation be tested by substituting the value of W for the point on the curve where $W = 280$ lbs., then $P = 46{\cdot}060$ lbs. Its actual value was 46 lbs.

Exercises. Find the equations to the following curves, drawing the curves where necessary :—

(a) To the curve of Example 3, page 68.
(b) ,, ,, Example 2 (a), page 72.
(c) ,, ,, Example 3, page 72.

Space Velocity and Acceleration Curves. In Example 2, page 72, on drawing curves to equations, the student was asked to draw curves to the equations (1) $v = gt$, (2) $s = \frac{1}{2}\, gt^2$, (3) $v^2 = 2\, gs$ where $g = 32$. He will recognize these as the well-known equations for falling bodies starting from rest, and we can use them here for illustrating some useful and important properties of curves representing space and velocity.

Fig. 37 shows the two curves drawn to the equations (1) and (2), just stated. In each case the equal distances 1, 2, 3, etc., along the base line represent one second of time. A is the curve to the equation $s = \frac{1}{2}\, gt^2$, and is therefore a space or displacement curve showing the space passed through at the end of the time of 1, 2, 3, etc., seconds. The space scale is $1''$ to 200 ft. B is the curve to the equation $v = g\,t$,

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and is therefore a velocity curve on a time base. It shows how the velocity increases with the time. We shall see that we could have obtained the curve B direct from the curve A, and that from B we can obtain a curve representing the acceleration of the body, the velocity of which is represented by the curve B.

To find the Velocity Curve from the Space Curve. If s = space passed through in any time t and v = average velocity during the time, then $s = vt$. Therefore $v = \frac{s}{t}$. Now looking at the curve A, we see that the space passed through at the end of the fifth second is represented by ab, and at the end of the sixth second by cd. Therefore the space passed through during the sixth second is represented by $cd - ab = ed$. That is:—

$$v = \frac{s}{t} = \frac{cd - ab}{ac} = \frac{ed}{be} = \frac{ed}{t} = ed,$$

since be represents one second.

This means, then, that the mean velocity during any single second is equal to the space passed through during the second. Therefore take the distance ed and set it up opposite the *middle* point m of the time intervals 5 and 6, since it represents the mean velocity during the time which is represented by the distance 5 to 6, or the length ac. It will be found to coincide with the curve B already drawn at the point n, and we know the curve B is the velocity curve for the space curve A. Care must be taken to see that the same scales are used, or that allowance is made for different scales. In Fig. 37 the scale for the velocity curve is doubled, therefore 1″ represents 100 ft. per second velocity. If the process is repeated for each other division, the points found should continue to coincide with the curve B.

We see, then, that by this method we can obtain a velocity curve from a space curve with a time base, and that, in fact, when the interval of time is one second with constant acceleration, the mean velocity during that second is equal to the space passed through during the second.

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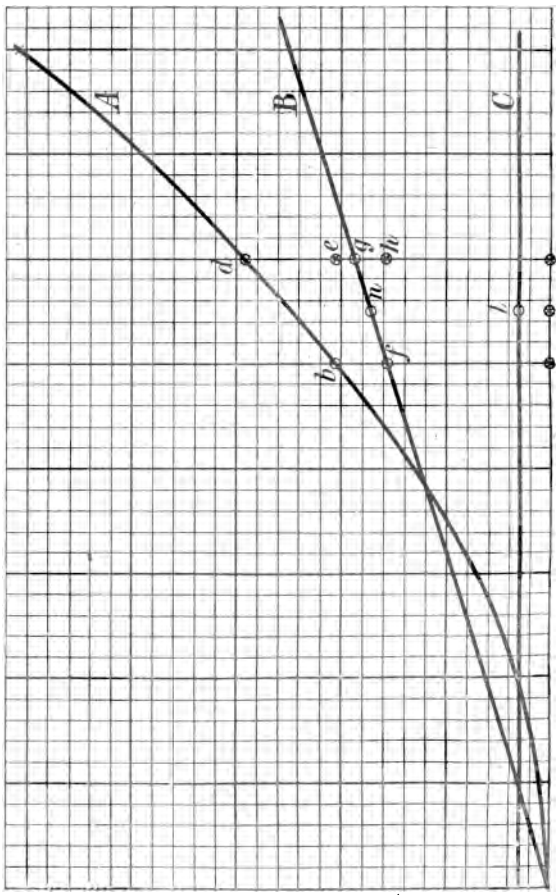

FIG. 87.
Space, Velocity and Acceleration Curves.

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To find the Acceleration Curve from the Velocity Curve. If V = initial velocity at beginning of time t, v = final velocity at end of time t, and a = acceleration, then $v = V + at$. Therefore $a = \frac{v - V}{t}$. In the curve B, the length af represents the velocity at the beginning of the fifth second, or the initial velocity V, and the length cg represents the velocity at the end of the fifth second, or the final velocity, then the acceleration during the second is—

$$a = \frac{v - V}{t} = \frac{cg - af}{ac} = \frac{gh}{t} = gh,$$

because ac represents one second.

Therefore the mean acceleration during the time t is represented by the length gh to the proper scale, that is, to the increase in velocity during the time. From m the middle point of ac set up ml equal to gh. Then l is a point in the acceleration curve. From what we know of the laws of falling bodies, and because the velocity as shown by the curve B increases uniformly, we know that the acceleration is constant; and that if we find the acceleration from other parts of the velocity curve, we shall get other distances equal to ml. The acceleration curve C will therefore be a straight line passing through l parallel to the base line. We are sure this is so, because in order to draw the space curve A we assumed a constant acceleration of g or 32 ft. per second per second. Notice that the distance gh represents increase in velocity per second in one second, which we know is acceleration. The velocity curve is not always a straight line, in which case the acceleration is not constant. In such cases the acceleration curve can be obtained from the velocity curve in the manner described, only the student should remember that the method then gives only approximately accurate results, as the values of the acceleration obtained from the velocity curve represent the mean acceleration during the interval of time, and assume that the velocity changes uniformly during that time, and that the acceleration is therefore constant during the time. The method is, however, sufficiently accurate for the examples likely to come within the reach of elementary students, and it is, after all, the principle of the thing which we wish him to grasp.

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To check Velocity and Acceleration Curves.

Velocity at beginning of fifth second $= 32 \times 5 = 160$ ft. per sec

„ end of fifth „ $= 32 \times 6 = 192$ „ „

$\therefore$ average velocity $= \frac{160 + 192}{2} = 176$ ft. per second.

Distance mn on curve $= 1{\cdot}76'' = 176$ ft. per second to scale taken.

Also $mn = de$ and de = space passed through during fifth second $= (\frac{1}{2} \times 32 \times 6^2) - (\frac{1}{2} \times 32 \times 5^2) = 176$ ft.

Mean acceleration during fifth second is shown on curve by distance $ml = 0{\cdot}32'' = 32$ ft. per second per second to scale used. But $ml = gh$ and $gh = yc - fa = 192 - 160 = 32$ ft.

It is very necessary to notice how we have simplified the matter, by considering the velocity and acceleration during the unit of time, *one second*. For, referring to the acceleration curve, we see that if ac did not represent one second, the acceleration would be $\frac{gh}{ac} = \frac{gh}{fh} = \tan \alpha$, where α is the angle gfh. That is, the acceleration is given by the tangent of the angle between the curve and the base line. This is the more general statement, and it is of course not quite so simple as the method we have adopted.

Exercise 1. Draw a space curve to the equation $s = \frac{1}{2} gt^2$ where $g = 32$, finding at least 8 points in the curve. From the space curve obtain a velocity curve, and from the velocity curve an acceleration curve. Check your working by calculation for at least one point in each curve.

Exercise 2. A piece of mechanism moves as follows:

Distance from starting-point at end of—

Time	.	1	...	2	...	3	...	4	...	5	...	6 secs.
Distance	.	0·42	...	1·68	...	3·24	...	4·68	...	5·7	...	6·06 ft.
Time	.	7	...	8	...	9	...	10	...	11	...	12 secs.
Distance	.	5·7	...	4·68	...	3·24	...	1·68	...	0·42	...	0 ft.

Draw the space curve and from it obtain a velocity curve, and from the velocity curve an acceleration curve. State the actual value of the velocity and acceleration at the end of the fourth second.

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CHAPTER VI.

ELASTICITY. STRESS AND STRAIN. HOOKE'S LAW. MODULUS OF ELASTICITY. TORSION OF WIRE.

Load and Pull. The experiments of this and the succeeding chapter are designed to illustrate some of the important properties of materials, such as indiarubber, steel, and wood, in the form of rods, wires, and beams, when acted upon by a force. The force applied to the material, whatever the effect it produces, is usually called the *load*. When the load is one which produces extension, it is known as a *pull*. A load may be applied slowly or suddenly, in which cases it is termed "*slowly applied load*" and "*suddenly applied load.*"

EXPERIMENT I. **To determine the effect produced upon a spring balance by a "slowly applied load" and a "suddenly applied load."**

APPARATUS. An ordinary extension spring balance reading to 20 lbs. by 4 ozs. For the suddenly applied load a strip of thin sheet brass about $1\frac{1}{4}''$ wide is bent around the front plate of the balance to form a spring clip, its upper edge being in contact with the pointer of the balance, so that the clip is pushed forward by the pointer, and remains in any position to which it is pushed. Weights of 2 lbs., 4 lbs., and 7 lbs.

Method. *Slowly applied load.* (1) Suspend the balance from some convenient position. (2) Note reading of pointer when unloaded. (3) Hang on the 2 lb. weight, taking care to do so very slowly and quietly. (4) Note the reading of the balance when it comes to rest. (5) Repeat the experiment with the 4 lb. weight and the 7 lb. weight.

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Suddenly applied load. (1) Fit the spring clip in position, so that its upper edge is in contact with the pointer when the balance is unloaded. (2) Take a 2 lb. weight, pass the ring of the weight over the hook of the balance, and hold the weight on the palm of the hand so that the ring just touches the hook of the balance, then *suddenly* withdraw the hand so that the whole weight comes on the balance as suddenly as possible. (3) Note the reading of the upper edge of the spring clip after the weight has come to rest. (4) Repeat the experiment for the load of 4 lbs. and 7 lbs., and record all the results.

RECORD OF EXPERIMENT. *Experiment on the effect of a slowly applied load and suddenly applied load on a spring balance.*

SLOWLY APPLIED LOAD.

	Load of 2 lbs.	4 lbs.	7 lbs.
Reading of balance before loading	0	0	0
„ „ after „	2	4	7
Difference due to load	2	4	7

SUDDENLY APPLIED LOAD.

	Load of 2 lbs.	4 lbs.	7 lbs.
Reading of balance before loading	0	0	0
„ „ after „	$3\frac{3}{4}$	$7\frac{3}{4}$	$13\frac{1}{2}$
Difference due to load	$3\frac{3}{4}$	$7\frac{3}{4}$	$13\frac{1}{2}$

What the Experiment teaches. By comparing the effect produced by the same load when applied slowly and applied suddenly, the student will see that the latter is nearly twice the former; that is, *a suddenly applied load produces twice the effect of the same load when slowly applied.* This indeed is exactly the conclusion obtained from theoretical considerations, the proof of which is outside the scope of the present work. This result is exceedingly important, and should be remembered in all future work. **It ought to impress upon the student the absolute necessity of applying loads to materials or structures as slowly and carefully as possible.**

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EXPERIMENT II. **To investigate the behaviour of a length of fine wire when loaded until it breaks.**

APPARATUS. (Fig. 38.) A length of ordinary metal wire about ·05″ or $\frac{1}{20}$″ diameter, free from twists or kinks, which pulls apart with a load of 120 to 160 lbs. A strong metal hook rigidly fixed to some convenient post or bracket about 6 ft. from the floor. A vernier gauge[1] for measuring the extension of the wire, reading to $\frac{1''}{1000}$, and with screws for fixing to the wire at points exactly 10″ apart, so that the extension is measured on a length of 10″. A strong scale-pan and weights.

FIG. 38.

Method. (1) Wind one end of the wire several times round the hook, and twist it around itself below the hook, so that it is not likely to slip when loaded. (2) Attach the scale-pan to the lower end in the same way, leaving a length of about 13″ between the two hooks. (3) Measure the diameter of the wire with a micrometer gauge. (4) See that the wire hangs quite straight, and free from bends or kinks. (5) Attach the vernier so that the distance between the points of attachment is exactly 10″. (6) Note the reading of the vernier. (7) Place weights in the scale-pan, increasing by 7 lbs. each time, and noting the reading of the vernier at each increase up to 36 lbs. (8) After reaching a load of about 36 lbs., slowly remove all the weights from the pan, and after waiting one or two minutes note the reading of the vernier again. (9) Replace the weights, and continue as before up to about 71 lbs. (10) Again remove the weights, and note the reading of the vernier. (11) Replace the weights, and continue as before, carefully watching the movement of the vernier as each weight is added, and noting the reading.

NOTE.—As the load increases, the wire will stretch more and more for the same addition of load, and the load should then be increased by only 4 lbs. each time, and a little later on by only 2 lbs. or 1 lb., until the wire breaks. The vernier should be carefully watched, so that the reading corresponding to the last load before breaking may be noted.

After completing the experiment, the results should be fully entered in the student's Laboratory note-book, and plotted on squared paper, taking loads along the base line, and extensions along the ordinates.

Note that the actual breaking load is the sum of the weight of the scale-pan, and the weights in the pan.

[1] An ordinary Chesterman's depth gauge lengthened by a strip of brass makes a capital vernier.

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RECORD OF EXPERIMENT. *Tensile test of iron wire.*

Diameter = 0·048″, area = 0·00181 sq. in., length between gauge points 10″, scale-pan weighs 1 lb.

No.	Load.	Reading.	Extension.	No.	Load.	Reading.	Extension.
	lbs.	inches.	inches.		lbs.	inches.	inches.
1	1	·520	·000	23	92	·540	·020
2	8	·521	·001	24	99	·541	·021
3	15	·524	·004	25	106	·544	·024
4	22	·526	·006	26	113	·546	·026
5	29	·527	·007	27	120	·548	·028
6	36	·528	·008	28	127	·551	·031
7	1	·520	000	29	1	·524	·004
8	15	524	·004	30	15	·528	·008
9	29	·527	·007	31	29	·531	·011
10	43	·529	·009	32	43	·534	·014
11	50	·531	·011	33	57	·537	·017
12	57	·532	012	34	71	·540	·020
13	64	·534	014	35	85	543	·023
14	71	·535	·015	36	99	·546	·026
15	1	·520	·000	37	113	·549	·029
16	15	·524	·004	38	120	·551	·031
17	29	·527	·007	39	127	·553	·033
18	43	·529	·009	40	134	·555	·035
19	57	·532	·012	41	141	·537	·037
20	71	·535	·015	42	148	·560	·040
21	78	·536	016	43	155	·564	·044
22	85	·538	·018	44	162	·568	·048
				broke	168	·574	·054

Elastic limit from curve = 94 lbs. = 23·2 tons per sq. in.
Breaking load = 168 lbs. = 41·3 tons per sq in.
Extension measured just before breaking = 0·054″ = 0·54%.

NOTE.—The wire used in this test is harder than ordinary iron bar, and the results must not be assumed as being similar to results from such bars. The breaking load is only approximately correct even with this wire, as it might have broken with any increase of only 1 lb. or less beyond the 168 lbs. Wires are usually very much stronger than the metal in ordinary bars and plates.

What the Experiment teaches. *Elastic Limit.* By looking at the curve (Fig. 39*A*) drawn through the extension points, we see that from the zero point up to about the point of 94 lbs. load the curve is practically a straight line, and that beyond that point it curves upwards more and more rapidly as the load increases.

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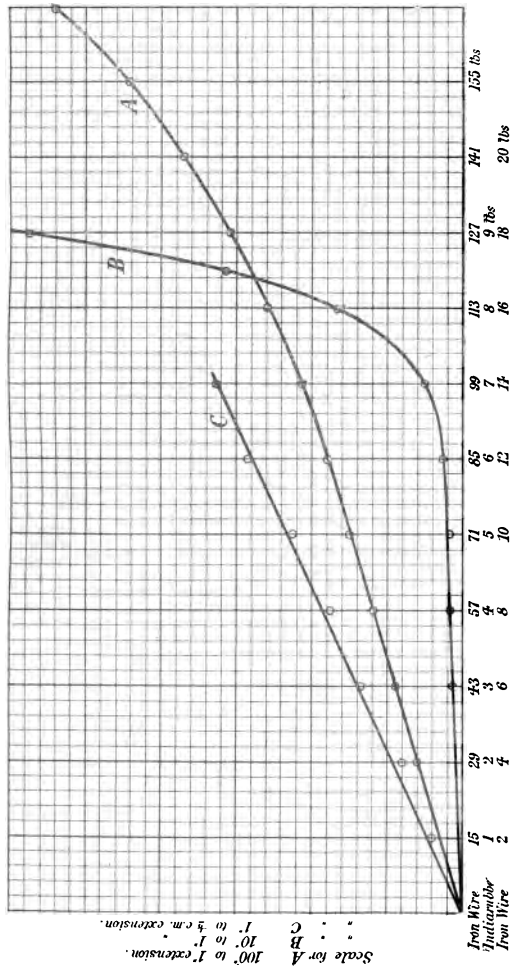
A
B
C
Scale for A 100" to 1" extension.
" " B 10" to 1" " "
" " C 1" to ½ c.m. extension.
Iron Wire
Indiarubber
Iron Wire
15 29 43 57 71 85 99 113 127 141 155 lbs
1 2 3 4 5 6 7 8 9 10 20 lbs
2 4 6 8 10 12 14 16 18

FIG. 30.

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From this we see that up to the load at which the curve leaves the straight line the extension is directly proportional to the load, and that after that point the extension increases more rapidly than the load. By referring to the table of results, it will be seen that when the load was removed at results No. 7 and 15 the wire returned to its original length, and that when the load was removed at No. 29 the wire did not return to its original length. We also see that the results Nos. 7 and 15 are for loads within the limits of the straight line, and that results No. 29 is for a load beyond the limit. Now these results are of the greatest importance, and deserve the fullest consideration. Up to the load of 94 lbs., that is to the point where the curve ceases to be a straight line, we say the wire is *elastic*, and the load at which the curve leaves the straight line is known as the *limit of elasticity*, or *elastic limit*. We have already seen that beyond the elastic limit the wire does not return to its original length when the load is removed, but remains permanently extended, and this extension is called a *permanent set*. The amount of the *set* is the difference between the extended length and the original length, and for the load of 127 lbs., result No. 28 and 29, this permanent set is 0·004″.

The readings of the vernier have measured the ultimate or maximum extension of the wire; and if the broken ends are examined, it will be seen that the wire has contracted in area at the point of breaking. It is scarcely possible to measure this reduction for so small a wire, but for larger sections it is always done. The difference between the original and the reduced area is known as the "*reduction in area*," and together with the extension is usually expressed as a per centage.

Owing to the limited experience of the student in experiments of this character, he will not be likely to grasp the full importance of these results; but there is no harm in saying that they are characteristic of all elastic materials, and are of universal application in connection with their testing and use. The same properties are shown much more clearly and forcibly when the material is a piece of mild steel bar about $1\frac{1}{2}$ or 2 square inches sectional area, and when it is pulled apart in a 50 or 100 ton testing machine.

The curve of Fig. 39 *B* is for a softer iron wire, 0·022″ diameter, breaking at 19 lbs., and extending 0·77 per cent.

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Stress and Strain. So far we have not considered the area of the wire, and we have spoken of the breaking force or pull as the load, meaning thereby the total load on the wire. Also by the extension of the wire we have meant the amount of its increase in length, without regard to the ratio of this extension to the original length of the wire. But it is evident that the fraction $\frac{\text{total load}}{\text{area of wire}}$ will vary in exactly the same way as the load merely, and also that the fraction $\frac{\text{extension}}{\text{original length}}$ will vary with the extension merely. Yet it is obviously more convenient to speak of load per unit of area and of the fractional increase in length, as we can then compare results with material of different sectional areas and different length. We therefore adopt the following:—

The quantity $\frac{\text{total load}}{\text{area}}$, or the load per unit of area, is called *Stress*. The ratio $\frac{\text{extension}}{\text{original length}}$, or the fractional increase in length, is called *Strain*. Therefore while the wire remained elastic and up to the elastic limit, *the strain produced was directly proportional to the stress which produced it.* This is true for all elastic material, and is known as Hooke's law of elasticity.

Hooke's law = Strain varies with the Stress.

EXPERIMENT III. **Test pieces of iron, steel, copper, and brass wire to breaking, measuring the extension on a length of 10″. Find the elastic limit, breaking stress in tons per square inch, and the maximum extension per cent. Plot curves for each wire, showing how the extension varies with the load.**

EXPERIMENT IV. **To find the relation between the stress and strain in a piece of indiarubber rod when loaded within the elastic limit.**

APPARATUS. A piece of solid round white rubber rod about $\frac{5}{8}$″ diameter and 3′ long. The ends can be turned

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round and tied to form loops. The rod is hung by one of the loops to a strong hook, as in Experiment II., and a scale-pan is attached to the other end. Two long darning-needles are pushed through the centre of the rubber rod at convenient distances apart (say 15″), and a metre scale is fixed just behind the needles, so that the distance between them may be easily read.

Method. (1) Measure the diameter of the rod with a micrometer gauge, taking at least three measurements in different places. (2) Note the reading of the needles. (3) Add weights to the scale-pan, increasing by 1 lb. each time, and taking the reading of the needles after each increase of load. *Care should be taken to take the reading of the needles after about the same length of time after loading in each case.* (4) Take readings with at least ten different loads. (5) After reaching the maximum load, remove the load by 1 lb. each time, and note the reading of the needles, thus securing a second set of readings for the same loads. (6) Arrange the results in column form, and find the extension at each load and for each increase of load. (7) Find the mean extension for the given load of 1 lb. (8) Express this load as *stress*, or lbs. per square inch, and the extension as *strain*, or fractional increase of length, and then find the value of the fraction $\frac{\text{stress}}{\text{strain}}$, the answer being in lbs. per square inch. (9) Plot the results on squared paper, setting off loads along the base line and extensions along the ordinates.

RECORD OF EXPERIMENT. *To find the ratio of the stress to the strain in a rubber rod when loaded within the elastic limit.*

Diameters of rod 0·613″, 0·613″, 0·608″, mean = 0·612″.

∴ area = 0·293 sq. in.

Length when loaded with scale-pan only 37·97 cm.

No.	Load. lbs.	Readings.		Length. Cm.	Extension from lgth. when unloaded.	Extension for each increase of load.	Readings Going Back.		Length. Cm.
		Bottom.	Top.				Bottom.	Top.	
1	0	60·15	22·18	37·97	0	0	60·3	22·2	38·1
2	1	60·2	22·2	38·0	0·03	0·03	60·5	22·25	38 25
3	2	60·5	22·22	38·28	0·31	0·28	60 95	22·3	38·65
4	3	60 82	22 26	38·56	0·59	0·28	61·4	22·4	39·0
5	4	61·15	22·3	38·85	0·88	0·29	61·8	22·5	39·3
6	5	61·5	22·4	39·1	1·13	0·25	62·2	22·6	39·6
7	6	61·85	22·45	39·4	1·43	0·30	62·6	22·75	39·85
8	7	62·15	22·55	39·6	1·63	0·20	63·0	22·8	40·2

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Mean extension for load of 1 lb. = ·235 cm. = ·0923″.

Original length = 37·97 cm. = 14·96″.

$$\therefore \text{ strain} = \frac{0{\cdot}0923}{14{\cdot}96} \qquad \text{stress} = \frac{1}{0{\cdot}293}$$

$$\therefore \frac{\text{stress}}{\text{strain}} = \frac{1 \times 14{\cdot}96}{{\cdot}0923 \times 0{\cdot}293} = 554 \text{ lbs. per sq. in.}$$

For curve see Fig. 39 *C*.

What this Experiment teaches. *Modulus of Elasticity.* No thoughtful student will have performed the experiment just described without observing some interesting things. In the first place it will be noticed that immediately the load is placed in the scale-pan the rubber rod extends somewhat rapidly, but that presently the rate of increase is lessened, until after an interval of one or two minutes it will appear to cease. If the student will now return to the experiment, and after taking the reading of the needles under the same conditions as before, he leaves the load on and takes the reading again at the end of a further three or four minutes, he will notice a very measurable increase of extension. The same will hold true at the end of a still further interval of time. This will suggest that the time during which a stress acts has some influence upon the strain it produces, and will show the necessity for taking the readings at the same time after loading in each case. The influence of time in the effect produced by a stress is indeed one of the most interesting and important questions in the behaviour of materials.

The result of taking off loads and noting the return readings will still further illustrate this. If the needles do not return to their first position, the student will find that they will probably do so if more time be allowed. If this is not the case, he will know the rubber has taken a "permanent set," and that its elasticity has been damaged. It is not always easy to get satisfactory results with indiarubber, but the author has found no difficulty with thick white rubber of good quality.

By referring to the curve (Fig. 39 *C*), it will be seen at once that the extension appears to increase directly with the load, as is shown by the fact that the curve which best passes through all the points is a straight line.

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Now since the extension varies directly with the load, it follows that the $\frac{\text{load}}{\text{extension}}$ is a constant. This is the same as saying that the $\frac{\text{stress}}{\text{strain}}$ is a constant, since stress $= \frac{\text{load}}{\text{area}}$, and strain $= \frac{\text{extension}}{\text{original length}}$, for, as already stated, the result of dividing each load by a constant, *i.e.*, the area, will not affect the ratio of the numbers representing the loads to one another, nor will the result of dividing each extension by a constant, *i.e.*, the original length, affect the ratio of the extensions; hence we see that $\frac{\text{stress}}{\text{strain}} =$ a constant.

This constant is termed the *Modulus of Elasticity*, or *Young's Modulus*, and is usually represented by the capital letter E, and expressed in units of force per unit of area.

$$\therefore \frac{\text{stress}}{\text{strain}} = E,$$

$$\text{or, stress} = E \times \text{strain},$$

$$\text{or, strain} = \frac{\text{stress}}{E}.$$

Hence we see that when we know the value of the modulus of elasticity, and either the stress or strain, we can find the strain or stress respectively. The numerical value of the Modulus of Elasticity may, therefore, be regarded as a number which expresses how many times a given stress is greater than the number representing the strain it produces, assuming the material to be elastic.

Now suppose it be possible to stretch a rod to double its original length, then the ratio $\frac{\text{extension}}{\text{original length}}$ or the strain $= 1$, and therefore the stress $= E$. Hence the modulus of elasticity in extension is often defined as the stress which will stretch a rod to twice its length, supposing it to remain elastic.

NOTE.—The above may also be read in connection with the wire test of Experiment II.

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EXPERIMENT V. **To find the modulus of elasticity for a wire.**

APPARATUS. The usual way of arranging this experiment is to take as long a length of wire as possible, owing to the fact that the extension within the elastic limit is so very small, not more than about $\frac{1}{1000}$th of its length for iron and steel, that the extension on a short length of wire is scarcely measurable by any simple means. In some laboratories arrangements are made for hanging wires from near the roof to the lowest floor, thus giving a length of 40 to 50 feet; but quite sufficient length can be obtained in an ordinary class-room if the upper end of the wire be fastened near the ceiling. It is very necessary that the upper end of the wire should be firmly fixed. There are two methods of easily measuring the extension; one is by connecting the lower end of the wire to a magnifying lever, the arms of which are in the ratio of 10 to 1, so that the long end of the lever moves through 10 times the distance of the short end, which is fixed to the wire. By allowing the lever to move across a strip of $\frac{1}{10}''$ squared paper, it is quite possible to read the extension accurately to $\frac{1}{200}''$ or $\frac{1}{500}''$. The second method is to hang two wires from the same support, one a measuring wire, and the other the wire for testing, the former being generally of smaller diameter than the latter. A vernier scale is used for reading the extension, the scale piece being attached to the measuring wire, and the vernier piece to the testing wire. The advantage of this method is, that if the support yields, the effect is felt equally by both wires, and the reading is not influenced. The vernier shown in Fig. 40 is made by adopting a Chesterman's depth gauge, the centre sliding piece being the vernier scale. The vernier can be read to $\frac{1}{100}''$ on one edge, and to $\frac{1}{10}$ mm. on

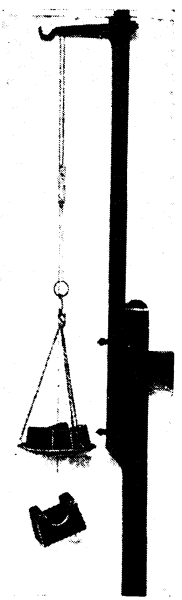

FIG. 40.

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the other edge. The wires are fixed by simple screw clips.

Care must be taken to ensure that the wire is quite free from kinks or bends, and that the diameter is carefully measured. The method of experimenting is exactly the same as for Experiment IV. with the rubber rod, except that the extension is read directly. At least ten readings should be taken with different loads; and the rate of increase of load should be sufficient to ensure a readable extension, and will, of course, depend upon whether the wire is of brass, copper, iron, or steel. The maximum load should not exceed about $\frac{1}{20}$ of the breaking load for copper, and $\frac{1}{10}$ for brass, iron, and steel. Convenient sized wires to use are Nos. 16 to 20, B.W.G.

RECORD OF EXPERIMENT. *Elastic modulus of tinned iron wire.* Diameter = 0·048 in., area = 0·00181 sq. in.

No.	Load.	Reading cm.	Extension cm.
1	0	1·10	0
2	4	1·15	·05
3	8	1·22	·12
4	12	1·27	·17
5	16	1·32	·22
6	20	1·37	·27
7	24	1·42	·32
8	28	1·46	·36
9	32	1·50	·40
10	36	1·54	·44
11	40	1·60	·50

The extension between

			cm.		cm.		cm.
20 lbs.	and	0 lbs.	= ·27	−	0	=	·27
24	,,	4 lbs.	= ·32	−	·05	=	·27
28	,,	8 lbs.	= ·36	−	·12	=	·24
32	,,	12 lbs.	= ·40	−	·17	=	·23
36	,,	16 lbs.	= ·44	−	·22	=	·22
40	,,	20 lbs.	= ·50	−	·27	=	·23

Mean extension for 20 lbs. = 0·23 cm. = 0·23 × 0·393″ = ·0905″.

The total length of wire = 185·75″.

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$$\text{Strain} = \frac{0{\cdot}0905}{185{\cdot}75} = {\cdot}0004875.$$

$$\text{The stress} = \frac{20}{0{\cdot}00181} \text{ lbs. per sq. in.} = 11050 \text{ lbs. per sq. in.}$$

$$\text{Modulus} = \frac{\text{stress}}{\text{strain}} = \frac{11050}{0{\cdot}0004875} = \frac{110{,}500{,}000}{4{\cdot}875} = 22{,}700{,}000$$

lbs. per sq. in.

NOTE.—The student must beware of assuming from this experiment that the value of E for iron is about 22,700,000 lbs. per sq. in. This is its value for the particular wire tested, and not necessarily for any other wire. The average value of E for iron bars is taken as being 27,000,000 lbs. per sq. in., and for mild steel 30,000,000 lbs. per sq. in.

EXPERIMENT VI. **Find the value of Young's Modulus of Elasticity in lbs. per square inch for wires of copper, brass, iron, steel.**

EXERCISES.

1. What do you understand by stress and strain respectively? If an iron rod, 50 ft. long, is lengthened by $\frac{1}{2}$ inch under the influence of a stress, what is the strain?

2. What is the modulus of elasticity of a substance? A round bar of iron, 12 ft. long and $1\frac{1}{2}$ square inches in sectional area. is held at one end, and pulled by a force until it stretches $\frac{1}{8}$ inch. Find the force, the modulus of elasticity being 30,000,000.

3. A rod 12 ft. long, with a uniform cross section of 4 square inches, is hung up by one end, and carries a weight of 20,000 lbs. at the other end; it is thereby stretched $\frac{1}{2}$ inch. Find the modulus of elasticity. What would have been the elongation if the length of the rod had been 20 ft. and the cross section 5 square inches?

4 An iron rod of 1 inch diameter and 12 feet in length stretches $\frac{3}{8\frac{1}{4}}$ inch under a load of 6 tons suspended at its extremity. Determine the stress, strain, and modulus of elasticity of the bar.

5 A wrought-iron tie-bar, $\frac{3}{4}$ inch in diameter, has a modulus of elasticity of 28,000,000 lbs. per square inch; its length is 23 inches. Find the load under which the bar will extend ·015 of an inch. Find also the stress per square inch.

6. Define "modulus of elasticity." If a wrought-iron bar of 1 square inch sectional area just breaks under a tensile stress of 60,000 lbs., what would be the area of the section of a tie-rod which would just support a load of 20 tons?

7. A bar 1 foot long is subjected to a tension of 1 ton per square inch of its sectional area. What is the nature of the resistance which it offers to elongation? If the elongation is $\frac{1}{10}''$, find the modulus of elasticity.

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Torsion, Shearing Stress, and Shear. One of the best examples of the action of torsion or twisting forces is seen in the working of ordinary shafting, where the transmission of energy by the shaft evidently produces a tendency to twist the shaft. It is possible to entirely separate the particles of a bar by fixing one end and applying a twisting force to the other end; and when this is done the particles separate due to a shearing stress, the strain produced being known as a shear strain.

The effect of a force producing twisting about a certain axis evidently depends upon the length of the arm at which it acts, or its *twisting moment.*

It is convenient to know how the amount of twist in a rod or shaft varies with the twisting moment, the length of the rod, and the diameter of the rod, and this can be conveniently and easily tested with thick metal wires.

EXPERIMENT VII. **To find how the twist of a wire depends upon the twisting moment.**

FIG 41.

APPARATUS. (Fig. 41.) The arrangement is shown in the illustration. The long board which should be fixed to the wall is about 10′ high, and $4\frac{1}{2}$″ by 2″, and is provided with a central groove in which movable brackets move, so that they can be fixed in any desired position. The brackets carry graduated dials marked in degrees. The upper arm is rigidly fixed, and carries a metal socket, into which one end of the wire to be tested can be tightly fixed by means of set screws. The lower end of the wire passes through the centre of the wheel boss, to which it is tightly fixed in the same way as the open end, and for thin wires a heavy weight is hung on the end to assist in keeping it taut. Cords and scale-pans are arranged as shown, thus acting as a "couple," and twisting the wire. Pointers are fixed on the wire, and move over the dials on the brackets.

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Method. (1) Fix the bracket with pointers at any convenient position, say near the top and near the bottom. (2) Take readings of the pointers without weights in the scale-pan, then add weights by 1 lb. each time and take the reading for each case. Record the results and plot them on squared paper, setting off the twisting force or twisting moment along the base line and the twist along the ordinates. (Note that as the arm of the couple is constant, the twisting moment is proportional to the twisting forces which are represented by the weights in the pans.)

RECORD OF EXPERIMENT. *Torsion of iron wire with different twisting moments.*

Iron wire = 0·251″ diameter. Length from fixed point at top to bottom pointer, 5′ – 0″. Scale-pans, 1 lb. each.

No.	Weights in Pan.	Reading of Pointer.	No.	Weights in Pan.	Reading of Pointer.
1	1 lb.	170°	11	11 lbs.	145°
2	2	167¼°	12	12	142½°
3	3	165¾°	13	13	139½°
4	4	162½°	14	14	137¼°
5	5	160°	15	15	134½°
6	6	158°	16	16	132¼°
7	7	155½°	17	17	129¾°
8	8	153°	18	18	127¾°
9	9	150¼°	19	19	124½°
10	10	148°	20	20	122¼°

For curve, see Fig. 42 *A*.

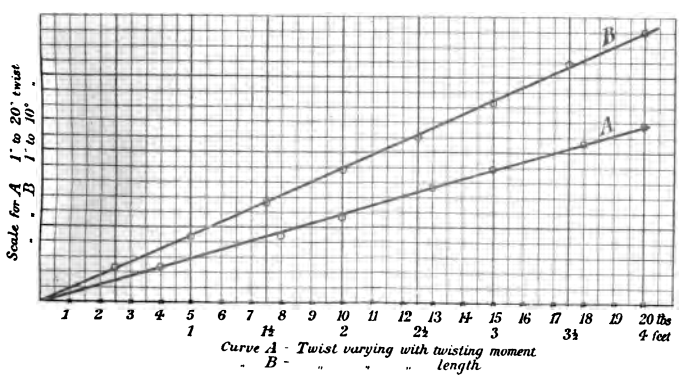
B
A
Scale for A 1″ to 20° twist
B 1″ to 10°
1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 lbs
1 1½ 2 2½ 3 3½ 4 feet
Curve A - Twist varying with twisting moment
B - " " " length

FIG. 42.

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What the Experiment teaches. The student should have no difficulty in seeing from this experiment that *the angle of twist is directly proportional to the twisting moment.* This is, of course, for shearing stresses within the elastic limit.

EXPERIMENT VIII. **To find how the angle of twist varies with the length of the wire.**

APPARATUS. As for Experiment VII.

Method. (1) The two brackets and pointers may be fixed in any desired position, say to divide the rod into three equal parts, or one of the brackets and pointers can be moved to different positions on the rod, say at distances of 1′, 2′, 3′, 4′, and 5′ from the fixed end, and the reading taken with the same twisting moment in each position. (2) The results should be plotted on squared paper as before.

RECORD OF EXPERIMENT. *How angle of twist of wire varies with length.*

Iron wire, 0·251″ diameter. Scale-pans, 1 lb. each. Total twisting force, 18 lbs. at 6·07″ arm; = 18 × 6·07 = 109·26 lbs. inches twisting moment.

No.	Length.	Reading of Pointers.		Difference. Angle of Twist.
		Top.	Bottom.	
1	5′ 0″	165¼°	125° full	45°
2	4′·6″	165¼°	130° bare	40°
3	4′·0″	165¼°	134°	36°
4	3·6″	165¼°	138¼°	31¾°
5	3·0″	165¼°	143½°	26½°
6	2′·6″	165¼°	148°	22°
7	2′·0″	165¼°	152½°	17½°
8	1′·6″	165¼°	157°	13°
9	1 0″	165¼°	161°	9°
10	– 6″	165¼°	165¼°	4¾°

For curve, see Fig. 42 *B*.

What the Experiment teaches. *This experiment should show that the angle of twist with a constant twisting moment varies directly with the length of the wire twisted.*

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EXPERIMENT IX. **To find how the angle of twist of a metal wire or rod varies with its diameter.**

This experiment will present no difficulty after Experiments VII. and VIII. Two or three wires of *the same material* and different diameters should be used, the twist for, say, the same two twisting moments being found in each case for the same length.

This experiment should verify the following law:—

The twist of a metal wire or rod of constant length and constant twisting moment varies inversely as the fourth power of the diameter of the wire.

EXERCISES.

8. If the twist in a wire $\frac{1}{4}''$ diameter at a point 5′ long from fixed end is 48° with a twisting moment of 120 lbs. inches, find the twist at a point 3′ from the fixed end with a twisting moment of 50 lbs. inches.

9. If the twist in a wire $\frac{1}{4}''$ diameter and 4′ long from the fixed point is 36° with a certain twisting moment, find the twist of the end of a wire $\frac{1}{8}''$ diameter and 5′ long from the fixed end.

10. If a wrought-iron shaft of 1″ diameter is twisted by the torsion of a load of 80 lbs. acting at the end of a 12″ lever, find the weight which, when applied to the end of the same lever, would twist equally a shaft of the same material, but 3″ diameter. State in general terms the reasoning by which you arrive at the result.

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CHAPTER VII.

STIFFNESS AND STRENGTH OF BEAMS.

To fully investigate the stresses and strains in beams either by mathematical or experimental methods would be quite beyond the limits of such a work as this. The student's knowledge of the way in which beams are used in practical construction, their various shapes, and methods of fixing and loading, will assure him of many of the difficulties in attempting anything but simple experiments. For example, we cannot very well experiment on the behaviour of a beam when subject to what is called a *live* load, such as that produced when a railway train passes over a bridge; nor can we very easily experiment with loads uniformly distributed over the whole length of the beam; nor, unless we possess a powerful testing machine, can we discover very much about the strength of large cast-iron or steel beams or girders, such as we see used so frequently in construction. All we can do in a simple and convenient manner is to experiment on small rectangular beams when the load is concentrated at one part, and from these results endeavour to deduce some of the laws which govern the two important properties of strength and stiffness.

Different Ways of Fixing and Loading Beams. There are six different ways of fixing or supporting, and loading simple beams, which are as follows:—

I. Beam fixed at one end and loaded at the other end.
II. Beam ,, ,, with load spread all over.
III. Beam supported at both ends and loaded at the middle.
IV. Beam ,, ,, with load spread all over.
V. Beam fixed at both ends and loaded at the middle.
VI. Beam ,, ,, with load spread all over.

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The above may be grouped into three pairs, depending upon how they are fixed or supported, each pair having either a concentrated load, or a load spread all over.

Cantilevers. *Beams fixed at one end only are termed* **cantilevers.**

Span of a Beam. *By the span of a beam is meant that part of its length which is unsupported, or the length of the beam between the supports, or, as in the case of a cantilever, the length of the beam which overhangs the support. For example, a beam which rests on two walls* 5 *feet apart is said to be of* 5 *feet span.*

Stiffness. *By the* **stiffness** *of a beam is meant its resistance to bending or deflection. A beam is said to be stiff when its deflection is very slight.*

Strength. *By the* **strength** *of a beam is meant its resistance to breaking.*

EXPERIMENT I. **To determine how the deflection of a beam varies with the load.**

APPARATUS. A rod of ordinary tool steel about $3\frac{1}{2}$ feet long, $\frac{1}{2}$ inch by $\frac{1}{4}$ inch, to act as the beam. It should rest upon steel or hard wood knife edges, which may be fixed to the wood blocks of the bench shown in Fig. 43. These blocks slide

FIG. 43.

between the upper and lower bearers of the bench, and are easily fixed in any required position by means of wood wedges.

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A short length of a metre scale divided into millimetres is screwed on the front of the bench, and a small light metal stirrup having a pointer fixed to it is used for the attachment of the scale-pan, and for reading the deflection. A scale-pan and weights.

Method. (1) Measure and note the breadth and depth of the beam. (2) Fix the supports at the required distance of 3 feet apart, and place the beam in position with the stirrup and scale-pan. (3) Place weights in the scale-pan until the weight of pan and stirrup and contained weights is some even weight as 1 lb. or 2 lbs. (4) Note the reading of the scale. (5) Increase the load by 4 lbs. each time, and note the deflection caused by each increase as shown by the scale. Continue the experiment until you have at least 10 results, taking care that the maximum load is well within the elastic limit of the beam (not to exceed 40 lbs.), and removing all the load at least twice to see if the beam returns to its first position. (6) Plot the results on squared paper, taking the load along the base line, and the deflections as ordinates. (7) Write down what you think the experiment shows as to how the deflection of the beam varies with its load.

RECORD OF EXPERIMENT. *To find how the deflection of a steel beam varies with the load.*

Beam 3 ft. span, $\frac{1}{4}$ inch deep, $\frac{1}{2}$ inch wide, supported at both ends and loaded at middle.

No.	Load.	Reading. Cm.	Deflection. Cm.
1	0	3·42	0
2	4	3·8	0·38
3	8	4·2	0·78
4	12	4·62	1·20
5	16	5·0	1·58
6	20	5·38	1·96
7	0	3·42	—
8	24	5·76	2·34
9	28	6·12	2·7
10	32	6·50	3·08
11	0	3·42	—
12	36	6·90	3·48
13	40	7·30	3·88

What the Experiment teaches. The student can scarcely have performed the experiment with any degree of care and

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thought without noticing that the relation between the deflection and the load is a very simple one. When the load is 8 lbs. the deflection was 0·78 cm.; when the load was doubled, or 16 lbs., the deflection was 1·58 cm., or almost exactly twice the first deflection; and when the load was 24 lbs., or three times as great, the deflection was 2·34 cm., or exactly three times. All this can only suggest that the deflection increases directly with the load, and the fact that a

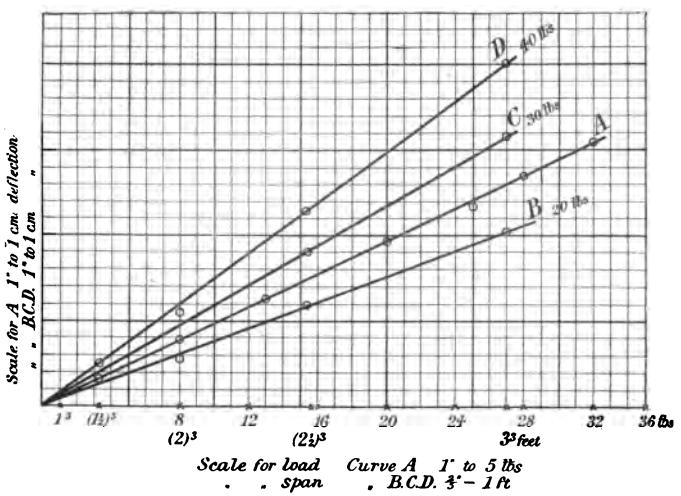
D 40 lbs
C 30 lbs
A
B 20 lbs
Scale for A 1" to 1 cm. deflection
" " B.C.D. 1" to 1 cm "
1³ (1½)³ 8 12 16 20 24 28 32 36 lbs
(2)³ (2½)³ 3³ feet
Scale for load Curve A 1" to 5 lbs
" " span " B.C.D. ¾" – 1 ft

FIG. 44.

straight line can be drawn through nearly all the points which represent the deflection, when plotted on squared paper, as seen in Fig. 44 *A*, only confirms this suggestion. We therefore conclude that the experiment teaches that the deflection varies directly with the load within the limits of the experiment.

The student will now naturally ask himself whether this is true for beams supported, or fixed in some other way and differently loaded. The best answer to this question will be found in trying the experiment with a beam fixed at one end only, and with a beam fixed at both ends. The fixing can

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easily be done by clamping the beam across the wood blocks already used, dispensing with the knife edge, and the load can be applied as before.[1] When this has been done, the same result will be obtained. Hence we are able to state the following law :—

Within the elastic limit of the material the deflection of a beam is directly proportional to its load.

EXPERIMENT II. **To determine how the deflection of a beam varies with its span.**

APPARATUS. Five rods of tool steel, each $\frac{1}{2}''$ wide and $\frac{1}{4}''$ deep, and $1\frac{1}{2}'$, $2'$, $2\frac{1}{2}'$, $3'$, and $3\frac{1}{2}'$ long, to act as beams of $1'$, $1\frac{1}{2}'$, $2'$, $2\frac{1}{2}'$, and $3'$ span respectively. Other apparatus as for Experiment I.

Method. (1) Find the deflection for each beam with the five different spans with loads of 20, 30, and 40 lbs., exactly as described in Experiment I. (2) Tabulate the results in parallel columns showing the deflection for the same load with the different spans as follows :—

DEFLECTION IN C.M.

No.	Load.	1 ft. span. Def.	1½ ft. span. Def.	2 ft. span. Def.	2½ ft. span. Def.	3 ft. span. Def.
1	20	0·08	0·35	0·53	1·20	2·05
2	30	0·11	0·45	0·79	1·81	3·13
3	40	0·15	0·55	1·09	2·32	4·07

What this Experiment teaches. The results will show the student at once that the relation between the deflection and the span is not quite a simple one, and that the deflection evidently increases very rapidly as the span is lengthened. Let us consider how we can discover the law by examining the results. We see that for a load of

20 lbs. the defl. on a 2′ span was $\frac{0\cdot53}{0\cdot08}=6\cdot6$ times greater than a 1′ span.

30 lbs. ,, 2′ ,, $\frac{0\cdot79}{0\cdot11}=7\cdot2$,, ,, 1′ ,,

40 lbs. ,, 2′ ,, $\frac{1\cdot09}{0\cdot15}=7\cdot3$,, ,, 1′ ,,

[1] It is very difficult to experiment with a load spread all over, and the author can only suggest that it might be done by using strips of lead of length equal to the span of the beam, and of some known or even weight.

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Much the same kind of figures are obtained by dividing the deflection with a 3′ span by the deflection for the same load on a $1\frac{1}{2}$′ span. The deflection then is evidently not proportional to the span, nor to the square of the span; indeed, it looks as though it might be proportional to the *cube* of the span, and is certainly more nearly this than any other simple relation. If this is correct, then a curve where the base line represents distances proportional to the cube of the span, and where the ordinates represent deflections, should be a straight line. Plot the results, therefore, on squared paper in this way, drawing a curve for each load as in Fig. 44, *B*, *C*, *D*.

The lines drawn are the ones which best pass through and fit the points, and ought to justify the student in thinking that he has verified the law connecting the deflection and span of beam, which is as follows:—

Within the elastic limit of the material the deflection of beams varies directly as the cube of the span.

NOTE.—The author does not pretend that the results given above are as accurate as ought to be obtained; but he has found it extremely difficult to obtain any more accurate ones. He has tried using beams made from carefully selected ash, and magnifying the extension by using a pointer moving over a graduated disc, but with practically no better results.

EXPERIMENT III. **To determine how the deflection of a beam varies with its breadth.**

APPARATUS. Five steel rods each $3\frac{1}{2}$′ long, one each of following sizes:—$\frac{1}{4}'' \times \frac{1}{4}''$, $\frac{3}{8}'' \times \frac{1}{4}''$, $\frac{1}{2}'' \times \frac{1}{4}''$, $\frac{5}{8}'' \times \frac{1}{4}$, and $\frac{3}{4}'' \times \frac{1}{4}''$. Other apparatus as for Experiments I. and II.

Method. (1) Find the deflection for each beam with loads of 11 and 18 lbs. and a span of 3′. The beams to be $\frac{1}{4}''$ deep in each case, and to be $\frac{1}{4}''$ wide for No. 1, $\frac{3}{8}''$ wide for No. 2, $\frac{1}{2}''$ wide for No. 3, $\frac{5}{8}''$ wide for No 4, and $\frac{3}{4}''$ wide for No. 5. (2) Plot the results on squared paper, taking the breadth along the base line and the deflection as ordinates. (3) Examine the results and the curves, and state what they teach as to the way in which the deflection varies with the breadth.

What the Experiment teaches. The student should have no difficulty in seeing from the results of this experiment that the effect of doubling the breadth of a beam is simply to halve

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its deflection, the result being to double the stiffness of the beam. Hence we have the following law :—

Within the elastic limit of the material the deflection of a beam is inversely proportional to the breadth.

EXPERIMENT IV. **To determine how the deflection of a beam varies with its depth.**

APPARATUS. The beams as used in Experiment III., and other apparatus as for Experiments I. and II.

Method. (1) Proceed exactly as in Experiment III. with loads of 11 lbs. and 18 lbs., the beam to be $\frac{1}{4}''$ deep for No. 1, $\frac{3}{8}''$ deep for No. 2, $\frac{1}{2}''$ deep for No. 3, $\frac{5}{8}''$ deep for No. 4, and $\frac{3}{4}''$ deep for No. 5. (2) Record the results in parallel columns and examine them exactly as for Experiment II., finding out by dividing the deflection for $\frac{1}{4}''$ depth into the deflection for $\frac{1}{2}''$ depth, and the deflection for $\frac{3}{4}''$ depth into the deflection for $\frac{3}{8}''$ depth. (3) If the results are worth anything at all, they should suggest that when the depth is doubled the deflection is only one-eighth; or in other words, that the deflection varies inversely with the cube of the depth. To test this, plot the results on squared paper, setting off the inverse of the depth cubed along the base line and the deflections along the ordinates. Do this for each load, and then see if a straight line can be drawn to fit the points.

What the Experiment teaches. The student should have no difficulty in seeing from the results of this experiment that the rule for the deflection of a beam varying with its depth is as follows :—

Within the elastic limit of the material the deflection of a beam is inversely proportional to the cube of the depth.

EXPERIMENT V. **To determine how the deflection of a beam depends upon how it is supported or fixed.**

APPARATUS. A bar of tool steel $2\frac{1}{2}'$ long, $\frac{1}{2}''$ wide, and $\frac{1}{4}''$ deep. Clamps for fixing ends to the wood blocks already described. Other apparatus as for former experiments.

Method. (1) Determine the deflection with loads of 7 and 14 lbs. on a span of two feet in each of the following cases: (*a*) when the beam is supported at both ends and loaded at the middle; (*b*) when the beam is fixed at one end and loaded at the other; (*c*) when the beam is fixed at

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both ends and loaded at the middle. NOTE.—The fixing must be firmly done by tightly clamping the end to one or two of the wood cross-pieces of the supports, which in their turn must be tightly wedged to the bench. In case (*b*) the stirrup can be prevented slipping off the beam by using a small clip or clamp. (2) Record the results in parallel columns, and express the deflection by numbers when the deflection for the beam, supported at each end and loaded in the centre, case 1, is represented by unity; that is, find how many times greater the deflection of the beam is when used as a cantilever than when supported both ends with a central load, and how many times less the deflection is when the beam is fixed both ends.

What this Experiment teaches. It is scarcely likely that the student will obtain results which are at all comparable with the values obtained from theoretical considerations, and he will probably find it better to be content with a very general conclusion. He will be sure to find that with a central load the deflection when the beam is fixed is less than when the ends are simply supported, and that the deflection is very much greater when the beam is fixed at one end only and loaded at the other. The actual relations are as follows:—

If the deflection of a beam supported at both ends and loaded at the centre is represented by 1, *then the deflection of the same beam when fixed at both ends is represented by* $\frac{1}{4}$, *and when fixed at one end and loaded at the other is represented by* 16.

Strength of Beams. The foregoing experiments, being on the stiffness of beams only, have been conducted with loads well within the elastic limit of the material. But in experimenting on the strength of beams, we require to load them until they break, and must therefore go beyond their limit of elasticity. Without a testing machine, it is only possible to do this conveniently with small beams of wood. Now the chief difficulty in using wood is that it varies so very much in its strength and properties, even though cut from the same part of the plank. You may cut two pieces of wood of the same size from the same part of a board, and although both may be free from knots or other faults, and may appear to be identical in character, yet when tested it is very doubtful if they will break at exactly the same load. Now if we want to compare the strength of two beams of different sizes in order to see how the size influences their strength, it is obvious that the material of

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which the beams are made should be of uniform strength, and the difficulty of ensuring this with wood beams makes it extremely hard to obtain results which accord with theory. But in spite of this, experiments on the strength of small wood beams are well worth performing, and cannot fail to impress upon the student certain general and important principles of very great value. The following experiments are suggested:—

EXPERIMENT VI. **To find how the strength of a beam varies with its breadth.**

APPARATUS AND METHOD. Use wood beams supported both ends and loaded in the middle. A span of 3′ will be convenient, and the beams may be (*a*) $\frac{3}{8}'' \times \frac{1}{2}''$, (*b*) $\frac{3}{8}'' \times \frac{3}{4}''$. Proceed exactly as in former experiments, continuing to add loads until the beam breaks, taking care to increase by small loads when near breaking.

RECORD OF EXPERIMENT. *The following are the results of experiments on pine beams performed as described above.*

(*a*) Beam $\frac{1}{2}''$ broad × $\frac{3}{8}''$ deep broke with central load of $11\frac{1}{4}$ lbs.
(*b*) „ $\frac{5}{8}''$ „ × $\frac{3}{8}''$ „ „ „ 14 „
(*c*) „ $\frac{3}{4}''$ „ × $\frac{3}{8}''$ „ „ „ $16\frac{1}{2}$ „

The breadths are as 1 : 1·25 : 1·5.

The strengths are as 1 : 1·244 : 1·46.

By theory the strengths should vary directly with the breadths.

The curve of Fig. 45 *A* shows the result of a test of a pine beam $\frac{5}{8}''$ broad × $\frac{3}{8}''$ deep, span 3′, breaking with a load of 13 lbs., and giving a maximum deflection of 13·6 cm. = 5·34″. Curve *B* is for a $\frac{1}{2}'' \times \frac{1}{2}''$ pine beam, 3′ span, breaking with a central load of 23 lbs., and giving a deflection of 8·3 cm. = 3·26″.

EXPERIMENT VII. **To find how the strength of a beam varies with its depth.**

APPARATUS AND METHOD.—Use beams, say, (*a*) $\frac{3}{8}'' \times \frac{3}{8}''$, (*b*) $\frac{3}{8}'' \times \frac{1}{2}''$, (*c*) $\frac{3}{8}'' \times \frac{5}{8}''$.

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RECORD OF EXPERIMENT. *The following are the results of experiments on two pine beams :—*

(*a*) Beam $\frac{1}{2}''$ deep $\times \frac{1}{2}''$ broad broke with a central load of 24 lbs.

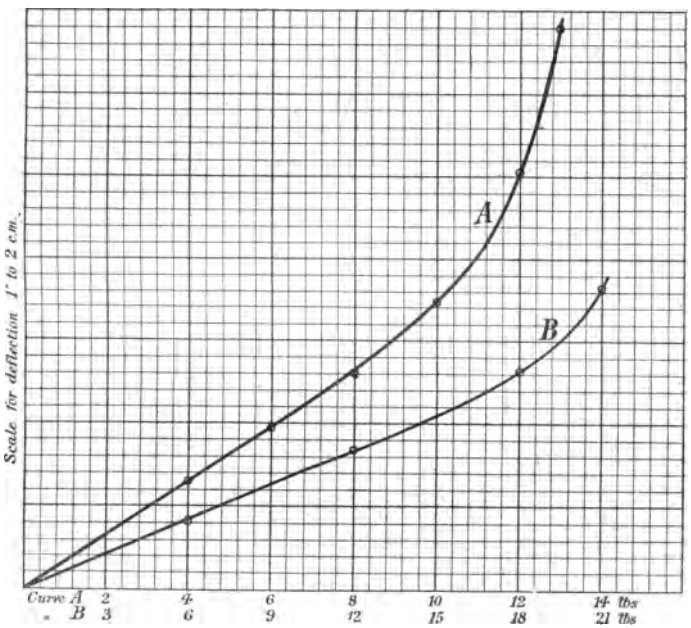

FIG. 45.

(*b*) Beam $\frac{3}{4}''$ deep $\times \frac{1}{2}''$ broad broke with a central load of 49 lbs.

The depths are as 1 : 1·5.

The strengths are as 1 : 2·04.

The depths squared are as 1 : 2·25.

By theory the strengths should vary directly with the depths squared.

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EXPERIMENT VIII. **To find how the strength of a beam depends upon how it is supported or fixed.**

APPARATUS AND METHOD. Use wood beams $\frac{1}{2}''$ wide $\times \frac{3}{8}''$ deep, span 1′, and test to breaking as follows: (*a*) Supported both ends and loaded at middle. (*b*) Fixed both ends and loaded at middle. (*c*) Fixed one end and loaded at other.

RECORD OF EXPERIMENTS. *Tests of pine beams as in Experiment VIII. gave the following results:—*

(*a*) Beam $\frac{1}{2}'' \times \frac{3}{8}''$ supported both ends and loaded in middle broke with load of 68 lbs.

(*b*) Beam $\frac{1}{2}'' \times \frac{3}{8}''$ fixed both ends and loaded in middle broke with load of 110 lbs.

(*c*) Beam $\frac{1}{2}'' \times \frac{3}{8}''$ fixed one end and loaded at other end broke with load of 14·5 lbs.

The strengths are as 4·7 : 7·6 : 1.

By theory they should be as 4 : 8 : 1.

How the Strength of a Beam varies with its Span. The student should scarcely require any experiments to show him how the strength is affected by the span. To double the span means simply to double the distance from the point about which the load is bending the beam; that is, to double the arm of the bending force, or to double the *bending moment*. Similarly, if we halve the span we halve the *bending moment*, and the beam is twice as strong. *The strength of a beam therefore varies inversely with its span.*

Stiffness. The deflection of a beam varies inversely with its breadth, inversely with the cube of its depth, and directly with the cube of its span.

If δ = deflection, b = breadth, d = depth, l = span,

$$\delta \propto \frac{l^3}{bd^3}$$

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Strength. The strength of a beam varies directly with its breadth, directly with the square of its depth, and inversely with its span.

If W = strength or breaking load,

$$W \propto \frac{bd^2}{l}$$

Method of Fixing and Loading Beams.

	Fixing and Loading.	Value of greatest bending moment.	Greatest bending moment at	* Relative values of Deflection.	* Relative values of Strength.
1	Beam fixed at one end and loaded at the other end	Load × span	Fixed end	16	$\frac{1}{4}$
2	Beam fixed at one end and loaded uniformly	Load × $\frac{1}{2}$ span	Fixed end	6	$\frac{1}{2}$
3	Beam supported at both ends with central load	Load × $\frac{1}{4}$ span	Middle	1	1
4	Beam supported at both ends and loaded uniformly	Load × $\frac{1}{8}$ span	Middle	$\frac{5}{8}$	2
5	Beam fixed at both ends with central load	Load × $\frac{1}{8}$ span	Middle and ends	$\frac{1}{4}$	2
6	Beam fixed at both ends and loaded uniformly	Load × $\frac{1}{12}$ span	Ends	$\frac{1}{8}$	3

EXERCISES.

1. A cantilever 1″ square and 12″ long will break with a load of 300 lbs. at free end. What will be breaking load at free end of a cantilever 1½″ wide, 2½″ deep, and 2′ long?

2. A wooden beam 12″ deep, 6″ wide, and 12′ long is embedded in a wall at one end. What weight will the beam carry at the outer end if

* These figures are taken from Perry's *Applied Mechanics*.

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the breaking weight of a beam 1′ long, 1″ broad, and 1″ deep, supported at the ends and loaded at the centre, is 500 lbs. ?

3. A wooden beam 12″ deep, 6″ broad, and 10′ long, is fixed at one end. What load will it carry evenly distributed if the breaking weight of a beam 1′ long, 1″ broad, and 1″ deep, supported at both ends and loaded at the centre, is 500 lbs. ?

4. A beam of timber, rectangular in transverse section, is 2″ broad, 3″ deep, and 4′ in length, and rests upon supports at its ends. The breaking load on the centre is 2000 lbs. What would have been the breaking weight if the beam had been 4″ deep, 2″ broad, and 4′ between the supports, but loaded at a distance of 1′ from the end ?

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CHAPTER VIII.

FRICTION.

THE student who has worked the experiments of the previous chapters should know something of what is meant by *Friction.* He will have found that one difficulty in the experiments on the triangle and polygon of forces was caused by what was called the *friction* of the pulleys, that the spring balances did not always work easily because of *friction*, and that one reason for using the conical points for the levers was that the lever might turn without *friction.* So that the intelligent and thoughtful student must have gained some general notion of what is meant by friction, although he will probably not be able to define it very clearly, or to say how it acts. The following few simple experiments will help to suggest other ideas about friction, and should assist the student in more difficult investigations into the laws of friction.

To Discover something of what Friction does.

EXPERIMENT I. (*a*) **Take a 7 lb. weight and place it on the top of a table. Push it gently in a horizontal direction, and then increase the push until the weight moves. Why is it that the gentle push is insufficient to move the weight, and why is a considerable push required to do so?**

Assuming the table-top to be horizontal, we know that the attraction of gravity does not offer any direct resistance to the movement of the weight; and we can only conclude that as soon as we push the weight some kind of force is called into play which acts against our push and resists movement.

(*b*) **Now place upon the table a sheet of ordinary glass having a smooth surface, and place the 7 lb. weight upon the**

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glass. Push the weight as before, increasing the push until the weight moves.

We notice at once that less push is required to move the weight than when it rested on the table, although it is exactly the same weight, and we therefore conclude that the force which resists the push, and acts to prevent motion, is less when the weight moves across the glass than when it moves across the wooden table-top.

EXPERIMENT II. (*a*) **Take a piece of straight board about 30″ long, 7″ wide, and $1\frac{1}{4}$″ thick, with one side planed to be fairly true and smooth. Place the board upon the table with the planed side upwards, and place a 7 lb. weight upon it at about its centre, then slowly raise one end of the board so that it becomes inclined at a greater and greater angle to the horizontal surface of the table. Notice that the weight remains at rest upon the board until, as the inclination increases, an angle is reached at which the weight commences to slide down the board.** (*b*) **Repeat the experiment with the sheet of glass upon the board.**

We know that the attraction of gravity upon the weight acts to make the weight slide down the board. But as the weight does not move, there must be a resisting force acting up the board, and this can only be caused by some force between the weight and the board. The fact that an angle of inclination is reached at which the weight does slide down the board shows that the component of the attraction of gravity down the board becomes greater than the resisting force acting up the board, and this should teach that the resisting force is limited, and does not increase indefinitely. The same lesson should have been taught by Experiments I. (*a*) and (*b*).

EXPERIMENT III. **Suspend a simple iron pulley, such as that shown in Fig. 46, from some convenient support. Arrange a piece of thick cord over the pulley, and tie a 7 lb. weight to each end of the cord. Call one of the cords *P* and the other *Q*, and arrange the weights to be at about the same level, so that there is an equal length of cord on each side of the pulley. Now pull the cord *P* downwards**

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with sufficient force to move the weights, and notice that a considerable pull is required before the weights will move. Move the weights back until they are again at about the same level. Tie a small extra weight to the cord *P*, taking care that this weight is small enough not to produce movement; a $\frac{1}{4}$ lb. weight will probably not be too much. Notice that a weight of 7 lbs. on the cord *Q* is supporting a weight of $7\frac{1}{4}$ lbs. on the cord *P*.

From this experiment we conclude that there must be some force acting which helps the smaller weight to support the larger one, and to resist movement of the weights. We notice also that the weights cannot move without turning the pulley, and we conclude that this other force must have something to do with the use of the pulley.

What these Experiments teach. If the student has thoughtfully and carefully carried out these simple experiments, he ought now to be able to see something of what friction does, and what we mean when we speak of friction. We say that friction acted to oppose the motion of the weight across the table and glass, that the friction between the weight and the inclined board prevented the weight from sliding down the board, and that the friction of the pulley helped the smaller weight to support the larger one.

He should have observed that the magnitude of the friction depends upon the surfaces in contact, as shown by the weight on wood or glass, and that there is evidently a limit to the amount of the friction. As he proceeds with experimental work, and uses his powers of observation in connection with the common occurrences of practical everyday life, he will see that friction is always found to be present between surfaces in contact, whatever the surfaces may be, if any attempt is made to produce relative motion, and that it generally shows itself as a force resisting motion. It does not always do this, as witness the common method of driving shafts by pulleys and belts, when the transmission of motion is entirely due to the friction between the pulleys and belts. He will see that friction can be diminished, although it can never be entirely got rid of, as for example, in the ball bearings of a cycle; and

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that although we often turn it to useful purpose, as in the case of "friction clutches" and certain forms of "friction driving gear," it more generally acts as a resisting force, reducing the useful work which a machine can do. Friction is as much the consequence of a natural law as is the attraction of gravity, and without it we could not walk about, or pull trains along on metal rails, or fasten wood with nails, or do many of the things which make life and work possible.

Friction and Limiting Friction. The amount of friction which can be called into play cannot exceed a certain limiting value. A distinction must then be drawn between the friction which exists between two surfaces in contact and the limiting value of such friction. It is stated among the general laws of friction that when there is no relative motion the amount of friction which is called into action is just as much as is necessary to prevent relative motion—and no more. If the force tending to cause motion is increased more friction is called into play, but there is always a limit to its amount, which is known as the limiting friction. The limiting friction between any two surfaces may, therefore, be defined as the greatest amount of friction which can exist between them if the conditions remain unchanged.

FIG. 46.

EXPERIMENT IV. **To investigate the friction of a simple pulley.**

APPARATUS. (Fig. 46.) Any form of simple pulley may be used for this experiment. For many reasons it is desirable to use one of the small pulleys used for experiments on the triangle of forces board, as the student can then discover the amount of their friction for different loads. But if a larger pulley is required, it is convenient to use a form of cast iron pulley known as a "gin block," a convenient size being 6″ or 8″ diameter. This pulley

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should be suspended some 5′ or 6′ from the floor, and a piece of ordinary sash cord or stout line should be placed over the pulley so that the ends are some 30″ from the floor. On one end hang a light scale-pan, and on the other a hook on which weights may be placed.

Method. (1) Weigh the scale-pan and hook. (2) Place extra weights in the scale-pan until, after giving the cord a slight pull to overcome the friction of rest, the pan moves down uniformly. (3) Repeat the experiment with weights of 7, 14, 21, 28, 35, 42, 49, and 56 lbs. on the hook. (4)[1] Weigh the sheave of the pulley, since its weight is evidently one of the causes of friction at the axle. (5) Plot the results on squared paper, marking the "load causing friction" along the base line and the "friction" along the ordinates, and find the equation to the curve drawn.

RECORD OF EXPERIMENT. *Friction of Simple Pulley.*

Weight of hook (P) = $4\frac{1}{2}$ ozs.
,, scale-pan (Q) = 17 ozs.
,, sheave of pulley = 4·81 lbs.

No.	Load P. lbs.	Load Q. lbs.	$Q-P$. = Friction.	$\frac{Q+P+\text{sheave.}}{\text{Friction.}}$
1	hook	—	—	—
2	7·28	7·845	0·565	·0283
3	14·28	15·25	0·97	·0284
4	21·28	22·7	1·42	·0291
5	28·28	30·18	1·91	·0301
6	35·28	37·44	2·16	·0278
7	42·28	44·95	2·67	·0290
8	49·28	52·43	3·15	·0296

The curve drawn setting off the total load and weight of sheave of pulley along the base line and friction along the ordinates

---

[1] NOTE.—Some difference of opinion exists as to what is the actual load on the axle of the pulley. It seems clear that it is equal to weight of cord and hook and scale-pan + weight of sheave of pulley + weights on hook + weights in scale-pan. It is usual to neglect the weight of the cord and of the hook if, as is generally the case, they represent only a very small fraction of the load. If P = weights on hook, Q = weights in scale-pan + weight of pan, w = weight of sheave of pulley, W = load on pulley, and F = friction,

then $F = Q - P$
$W = P + Q + w$.

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gave a straight line as the line best passing through all the points, thus showing that the friction varies with the load.

What the Experiment teaches. The student should have no difficulty in seeing from this experiment that the *friction would appear to increase with the load*, so that the ratio of the friction to the total load approaches a constant. This is a very important law, and should be remembered in all following experiments. An experiment with a smaller and better balanced pulley will probably give even more accurate results than those recorded above.

To discover some of the Laws of Friction between plane surfaces in contact. As the same piece of apparatus can be used for several experiments, it will be as well to explain its construction and use before describing the experiments themselves.

APPARATUS *The Friction Board.* (Fig. 47.) A piece of mahogany, or pine board, or any convenient wood, about 30″ long, 7″ wide, and $1\frac{1}{4}$′ thick. Let it be quite dry, and free from knots, splits, and twists. One side is planed up to make as smooth and flat a plane surface as possible, and to the other side are screwed two narrow cross-pieces, one at each end, to form supports for the board, so that when they rest upon a flat surface there may be no rocking. At the centre of one end is attached a light easily running grooved pulley, so that the top

FIG. 47.

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point of the groove is about $\frac{5}{8}''$ above the planed surface of the board.

Sliders. These consist of pieces of board of the same or different wood, and as carefully selected as the friction board, each $1\frac{1}{4}''$ or $1\frac{1}{2}''$ thick, and about $9'' \times 6''$. One side of each slider is planed up as true and smooth as possible, and a cross-piece about $\frac{3}{4}''$ wide and $\frac{5}{8}''$ thick is screwed on this side close to one of the edges, *and parallel to the direction of the fibres of the slider.* A small metal hook or eye is screwed into the outer edge of the cross-piece on its centre line, and the same distance from the planed face of the slider as the top point of the groove in the pulley is from the surface of the board. A strong silk cord about $\frac{1}{16}''$ to $\frac{3}{32}''$ diameter is tied to the eye of the slider, and a light, small scale-pan to the other end, the length of the cord being adjusted so that when the slider is near the end of the friction board farther from the pulley the scale-pan just hangs clear of the pulley.

Two sliders may be cut from the same board, the face of each to be of equal flatness and finish, and one to be half the area of the other (say, one $9'' \times 6''$ and the other $4\frac{1}{2}'' \times 6''$), or, better still, cut the slider from a board $1\frac{1}{2}''$ or $2''$ thick, and remove part of one face (as shown in Fig. 48) so that the area of bottom face is half that of the upper one. A light wood cover must be used to protect the upper face of the slider (as shown in Fig. 47).

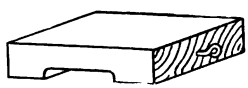

FIG. 48.

NOTE.—It is not essential that the fibres of the slider should be at right angles to those of the board, and, indeed, it is exceedingly instructive to experiment first with the fibres of slider and board parallel, and then with fibres at right angles, but rather better results are obtained when arranged as suggested.

EXPERIMENT V. **To find the force required to overcome the limiting friction between wood and wood with a given pressure.**

Method. (1) Place the friction board upon a table so that the pulley overhangs to clear the pan. Test whether the surface of the board is horizontal by using a spirit level, and, if not, make it so by packing the ends. In order to keep it from moving, clamp it to the table, or place a

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14 or 28 lb. weight upon the back end of the board. Oil the pulley, and see that it turns quite freely. (2) Weigh the slider and the pan, and record in your note-book how much they weigh. (3) Place the slider at the back of the friction board, arrange the cord over the pulley, and tie on the pan. (4) Load the slider with a 14 lb. weight, and place weights in the pan until the slider starts from rest and moves along the board. (5) Record in your note-book the weights on the slider and in the pan, and your observations on how the slider moved after it started. (6) Repeat the experiment with loads on the slider of weights of 21 and 28 lbs. (7) Repeat the experiment with the same weights on the slider, but instead of putting weights in the pan until the slider begins to move, place in only sufficient weights to keep the slider moving with a steady uniform motion after starting it by a slight push, or by a smart thump on the end of the friction board. A little practice will be required to do this satisfactorily. If the slider moves with an increasing velocity after being started, some of the weights in the pan must be removed, while if it moves in jerks and occasionally sticks, more weights should be added to the pan. By a little adjustment of this kind it will be found quite easy to ensure that, after being started, the slider shall move with uniform velocity for the whole length of the board.

Recording the Results of the Experiment. The experiment should now be entered in the student's Laboratory book somewhat as follows, and a simple diagrammatic sketch of the apparatus used should be made in the book:—*Experiment on force required to overcome the friction between pine and mahogany.*

Pine slider 9″ × 6″, weight 1 lb.
Weight of pan 2½ ozs.

No.	Slider and weights.	Force Required to Move Slider.	
		Slider starting from rest.	Slider started by hand.
		lbs. weight.	lbs. weight.
1	8 lbs.	5·406	3·906
2	11 ,,	10·656	6·656
3	22 ,,	13·656	8·656

What the Experiment teaches. The results just recorded are of an actual experiment, and if those obtained by the student are at all similar, as they should be, he should be able to state the following: *first*, that the force required to start the slider from rest is much greater than the force required to keep

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it moving with uniform velocity after it has been started by hand; *second*, that when the slider is started from rest by the weights in the pan, it moves with an increasing velocity, showing that the force causing motion is greater than that necessary to just overcome the friction between the slider and board, and that part of it is imparting an acceleration to the moving parts.

Friction of Rest. The friction which resists motion between bodies at rest is termed "Friction of Rest" or "Statical Friction," and is thus distinguished from the friction of bodies when moving, which is known as "Friction of Motion" or "Kinetic Friction." In the experiment just concluded the friction of rest is greater than the friction of motion, and the student should observe whether this applies generally. *He should note that in the remaining experiments on friction it is the friction of motion which is sought for, and so that the slides should be started by hand in all cases.*

EXPERIMENT VI. **To find the limiting friction between wood and wood with different pressures between the surfaces in contact, and to find how the limiting friction varies with the pressure.**

Method. (1) Proceed exactly as explained in Experiment V., for friction of motion, using weights on the slider of 4, 7, 11, 14, 18, 21, 25, 28, 32 lbs., letting the first experiment be with the weight of the slider only, and care being taken that the motion of the slider is uniform.[1] (2) Record your results as you obtain them in column form as follows: Col. 1, No. of Experiment; Col. 2, Slider and Weights; Col. 3, Force causing motion (pan and weights). (3) After completing the experiments with the loads given, find the result obtained by dividing the "force causing motion" by the weight of the "slider and weights" for each experiment, and record the result in a 4th column of your note-book. (N.B. — *This should not be done until the actual experiment with the apparatus has been concluded.*)

[1] N.B.—The student will probably find that with large weights on the slider, a slight difference in the weights in the pan will not cause any appreciable difference in the motion of the slider; for example, in a certain experiment it was found that the slider moved with any force from 3 lbs. 10¼ ozs. to 3 lbs. 11¼ ozs. when the weight on the slider was one of 14 lbs., and with any force from 6 lbs. 4¼ ozs. to 6 lbs. 6¼ ozs. when the weight was 28 lbs. This is exactly one of the things which the experiment is intended to teach, as showing some of the features of friction, and the difficulty of obtaining absolutely uniform results. The greatest difference is rather less than 2%.

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Normal Force. The total weight of the slider and of the weights upon it is the measure of the normal force between the slider and the friction board. We shall call this in future the "normal force between the surfaces in contact," or simply the "*normal force.*" In practical text-books it is usually spoken of as the "pressure," but it seems better to use this word to mean stress per unit of area.

Limiting Friction. As the board is horizontal, the only resistance to motion is that caused by the *friction* between the surface in contact, and uniform motion is obtained as soon as the pull in the cord is equal to this friction, after exerting a slight extra pull or push to start the slider. But the pull in the cord is equal to the sum of the weights in the pan and the weight of the pan and neglecting the friction of the pulley; this is equal to the *limiting friction* between the surfaces in contact.

We shall therefore call this in future the *Friction.*

Record of Experiment. *To find the limiting friction between wood and wood with different pressures between the surfaces in contact, and to find how the friction varies with the normal force.*

Board and slider of mahogany.
Fibres of board and slider parallel.
Slider $6{\cdot}4'' \times 6''$, weight $= 0{\cdot}78$ lbs.
Weight of pan $= 0{\cdot}156$ lbs.

No.	Normal Force.	Friction.	Friction. / Normal Force.
	lbs. weight.	lbs. weight.	
1	0·78	0·234	0·299
2	4·78	1·531	·320
3	7·78	2·125	·273
4	11·78	3·218	·273
5	14·78	4·093	·277
6	18·78	5 218	·277
7	21·78	5·906	·271
8	25·78	6·781	·263
9	28·78	7·656	·266
10	32·78	8·781	·267

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$$\text{Mean value of } \frac{\text{Friction}}{\text{Normal Force}} = 0{\cdot}278.$$

N.B.—A curve should be drawn showing the relation of the friction to the normal force, taking the value of the friction along the ordinates, and the normal force along the base line.

What the Experiment teaches. *Coefficient of Friction.* By observing the figures in the last column of the table of results, we observe that they are almost exactly the same, and that the value of $\frac{\text{Limiting Friction}}{\text{Normal Force}}$ appears to be a constant. We are indeed justified in inferring that but for inaccuracies in the experiment this would be so exactly, especially as we notice that the friction increases in direct proportion to the normal force.

$$\therefore \frac{\text{Limiting Friction}}{\text{Normal Force}} = \text{a constant.}$$

This constant is termed the "coefficient of friction," and is usually represented by the Greek letter μ (mu).

If F = limiting friction and P = normal force, then

$$(1)\ \frac{F}{P} = \mu \qquad (2)\ F = \mu P.$$

The student should now add to his record of the experiment in his Laboratory book a note to the effect that the experiment shows that the friction varies directly with the normal force, so that the ratio of the first to the second is a constant, which is called the *coefficient of friction.*

The use of the coefficient of friction will be shown by working the following examples:—

EXERCISES.

1. A locomotive draws a load of 200 tons. Find the pull needed at a constant speed if the friction is ·05 of the load.

2. What is the coefficient of friction, and how is it ascertained? There is an inclined plane of 1 ft. vertical to 10 ft. horizontal; what work is done in moving 700 lbs. through 5 ft. along the plane, the coefficient of friction being ·08?

3. What is friction? What is meant by limiting friction by sliding friction, and by the coefficient of friction? A weight of 5 cwt. resting

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on a horizontal plane requires a horizontal force of 100 lbs. to move it against friction. What, in that case, is the value of the coefficient of friction?

4. A small metal planing machine, the table of which weighs 1 cwt., makes six backward and six forward strokes, each of 4½ ft. in a minute, and the coefficient of friction between the sliding surfaces is 0·07. What is the work performed in foot-pounds per minute in moving the table?

5. How would you experimentally determine the nature of the friction between clean smooth surfaces, say of oak, and what sort of law would you expect to find?

6. A locomotive with three pairs of wheels coupled weighs 35 tons, the coefficient of friction between wheels and rail is 0·18. Find the greatest pull which the engine can exert in propelling itself and a train. What is the total weight of itself and train, which it can draw up an incline of 1 in 100, if the resistance to motion is 12 lbs. per ton on the level?

RECORD OF EXPERIMENT. *Friction of pine on pine with grain of each parallel to line of motion.*

Weight of slider = 1 lb.; weight of pan = 1 lb.

Each experiment performed twice, and the curve of friction drawn to mean of the two results. The value of the friction is also given as corrected from curve.

FRICTION.

No. of Exp.	Normal Force. Slider and Weights.	1st Exp. Pan and Weights.	2nd Exp. Pan and Weights.	Mean.	Mean corrected from curve.	$\mu = \frac{F}{P}$
	lbs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	
1 30	4	1 0	0 13¼	0 14⅝	0 14⅝	0·2285
2 29	5	1 2	1 0¼	1 1⅛	1 1½	0·2187
3 28	6	1 4	1 5¼	1 4⅝	1 4½	0·2135
4 27	7	1 7	1 6¼	1 6⅝	1 7⅜	0·2086
5 26	8	1 12	1 8¾	1 10⅜	1 10¼	0·2050
6 25	9	2 0	1 13½	1 14¾	1 13	0·2014
7 24	10	2 1½	1 15½	2 0½	2 0	0·2000
8 23	11	2 3¼	2 2	2 2⅝	2 2¾	0 1970
9 22	12	2 6	2 5	2 5½	2 5½	0·1953
10 21	13	2 8½	2 8½	2 8½	2 8½	0·1947
11 20	14	2 11¾	2 9¾	2 10¾	2 11⅝	0·1943
12 19	15	2 15¾	2 14¼	2 15	2 14¼	0·1927
13 18	16	3 2	3 0¼	3 1⅛	3 1⅛	0·1919
14 17	17	3 3¾	3 2¾	3 3¼	3 3¼	0·1911
15 16	18	3 6¾	3 6¾	3 6¾	3 6¾	0·1901

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EXPERIMENT VII. **To find how friction depends upon the area of the surfaces in contact, using wood on wood.**

APPARATUS. The same as for Experiment V., using a slider as in Fig. 48, with one face twice the area of the other.

Method. (1) Weigh the slider and the pan. (2) Find the friction for ten different loads on the slider, using weights of 0, 4, 7, 11, 14, 18, 21, 25, and 28 lbs., and using the large face of the slider exactly as in Experiment VI. (3) Repeat the experiment with the same weights, using the small face of the slider.

RECORD OF EXPERIMENT. *To find how friction depends upon the area of the surfaces in contact, using wood on wood.*

Board of mahogany.
Large face of slider 9″ × 6″, area = 54 sq. in., weight = 1 lb.
Small ,, ,, $4\frac{1}{2}$″ × 6″ ,, = 27 ,, ,, = 1 lb.
Slider of pine.
Weight of pan = 2·5 ozs.

No.	Normal Force.	Friction.		Coefficient.	
		Large Slider.	Small Slider.	Large Slider.	Small Slider.
	lbs. weight.	lbs. weight.	lbs. weight.		
1	1	0·25	0·343	0·250	0·343
2	5	1·40	1·656	·281	·331
3	8	2·219	2 406	·277	·300
4	12	3·218	3·406	·268	·283
5	15	4·031	4·281	·268	·285
6	19	5 093	5·156	·268	·271
7	22	6·406	6·406	·291	·291
8	26	7·781	6·906	·299	·265
9	29	8·656	7·906	·298	·272
			Mean	0·277	0·293

What the Experiment teaches. The results of the actual experiment performed by the student are not likely to differ greatly from those recorded above. At first sight the figures do not seem to convey any very definite idea as to how the friction has been affected by the change of area, yet further consideration would surely suggest the conclusion that although the area in one case is twice that in the other, the friction is

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practically the same. In Experiment No. 7 the friction is the same for both sliders; in Nos. 5 and 6 it does not differ by more than 3 per cent., and we have already seen that the magnitude of the friction with large weights, as found by experiment, may vary by 2 per cent. The conclusion can be further tested by experimenting in a somewhat different way, as follows:—

EXPERIMENT VIII. **To determine the friction with weights of 7, 14, 21, and 28 lbs., using a slider of pine on a mahogany board; and to determine whether the friction is the same with a slider of half the area under the same normal force.**

Method. (1) Proceed exactly as before to find the friction with the large face of the slider and the weights stated. Take special care to note the rate of motion of the slider in each case, and if possible to measure its velocity by noting the time taken to move between two fixed marks. (2) Use the small face of the slider with the same weights, so that the normal pressure is the same as with the large slider, and place the same weights in the pan as required for the corresponding normal force with large slider. Then start the slider as usual, and see if its velocity is the same as with the large slider; if it is less, add weights to the pan; if greater, remove weights, thus adjusting the pull on the slider until it moves as nearly as can be measured identically with the corresponding experiment with the large slider. (3) Enter the results as for Experiment VII.

By proceeding as described above, it will probably be found that the difference between the friction for the large and small slider is much less than when experimenting as in Experiment VII., and that the value of μ is practically constant. This being so, our conclusion is strengthened that the friction between wood surfaces in contact is independent of their area. We will now deal with other surfaces.

EXPERIMENT IX. **To find the friction between cast iron and cast iron with different normal force, and to determine whether the friction depends upon the areas of the surfaces in contact.**

APPARATUS. (Fig. 49.) The apparatus is the same as in previous experiments, except that a plate of cast iron about

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3 ft. × 7″ and 1¼″ thick is substituted for the board, and that the slider is of cast iron of shape shown in Fig. 49.

FIG. 49.

Method. (1) Proceed exactly as in Experiment VI., using the large surface of the slider with at least ten different loads. (NOTE.—The upper face of the slider must be covered by a board to protect it from any injury from the weights used.) (2) Perform experiments with the same loads, using the small face of the slider, proceeding in each case either as in Experiment VII. or Experiment VIII., or with both. (3) Record the results as in Experiment VI.

An experiment actually performed showed a mean value of μ with the large surface of 0·224, and with the small surface of 0·206, the difference between the friction for similar pressures being slightly greater than with pine or mahogany. (Experiment VI.) But it will be strange if the student's experiments are not sufficiently uniform to further confirm the conclusion that friction is independent of the area of the surfaces in contact. He will also notice the difference in the coefficient for wood on wood and for cast iron on cast iron. If he tries wood on wood with very small areas under great pressures, he will find that the results are not so uniform, the reason being that the surfaces in contact become more closely squeezed together, so that their condition of smoothness is changed.

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RECORD OF EXPERIMENT. *To find Coefficient of Friction for gun-metal on cast iron dry.*

Weight of slider = 14 lbs. Weight of pan = 9 ozs.

No.	Slider and Weights.	1st Exp. Pan and Weights.	2nd Exp. Pan and Weights.	Mean.	Mean corrected from Curve.	$\mu = \frac{F}{P}$
	lbs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	
1 14	14	3 10	4 4	3 15	3 15	0·2813
2 13	21	5 8	6 2	5 13	5 14	0·2793
3 12	28	7 9	7 15	7 12	7 11	0 2746
4 11	35	9 0	9 4	9 2	9 10	0·2750
5 10	42	11 1	11 12	11 6½	11 8	0·2738
6 9	49	13 12	13 14	13 13	13 6	0·2730
7 8	56	15 6	15 6	15 6	15 6	0 2746

EXPERIMENT X. **To find the friction between cast iron and cast iron when the rubbing surfaces are lubricated with ordinary machine oil.**

APPARATUS. The same as in Experiment IX.

Method. (1) Pour some oil on the face of the friction plate, spread it well over the surface, place on the slider, and move it backwards and forwards over the plate to spread the oil uniformly over the surface. (2) Proceed exactly as in Experiment IX., using the same weights on the slider. (The experiment may be done with the large face of the slider only, or with both large and small faces.) (3) Record the results in your Laboratory note-book, and work out the coefficient in each case.

The student should find that in this experiment he is face to face with entirely new conditions. The slider will seem to be floating on a film of oil, and will move so easily that the slightest force will change its position. Only a very slight push will be necessary to overcome the friction of rest, and a comparatively small force will keep it in motion. Then as more and more weights are added to the slider these conditions become less marked, a result which the student should readily trace to the greater normal force squeezing out the oil from between the rubbing surfaces, so that we more nearly approach the conditions of Experiment IX. He will also probably observe that the same weights in the pan will move the slider with apparent

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similarity of motion when the load on the slider is different; as, for example, in an actual experiment the same weights in the pan moved the slider when loaded with weights of 42 lbs. as when loaded with weights of 28 lbs. This and other variations from former results are sure to occur, and, apart from teaching the important fact of the lessening of the friction due to lubrication, they ought to lead the student to consider whether the laws of lubricated friction are quite as simple as when the rubbing surfaces are dry. If opportunities permit, he should experiment further with different kinds of lubricants and with greater pressures, and should make himself acquainted with what has been done by other investigators.

RECORD OF EXPERIMENT. *Experiment on the friction of cast iron on cast iron with surfaces of different areas and under different pressures*—(a) *with surfaces dry*, (b) *with surfaces lubricated.*

Slider, large face = 6·9″ × 6·9″.
small ,, = 3·45″ × 6·9″.
Weight = 9·25 lbs., weight of pan = 0·5 lbs.

No.	Normal Force.	Surfaces Dry.				Surfaces Lubricated.			
		Large Slider.		Small Slider.		Large Slider.		Small Slider.	
	lbs. wt.	Friction lbs. wt.	μ	Friction lbs. wt.	μ	Friction lbs. wt.	μ	Friction lbs wt.	μ
1	23·25	5·5	0·236	5·0	0·215	1·375	0·0591	0·875	0 0376
2	27·25	6·25	·229	5·75	·211	1·68	·0616	1·25	·0458
3	30·25	6·5	·214	6·0	·198	1·75	0578	1·50	·0495
4	34·25	7·25	·211	7 0	·204	1 87	·0546	1·625	0474
5	37·25	7·5	·201	7·0	·188	2·25	·0604	2·0	·0536
6	41·25	8·25	200	7 5	·181	—	—	—	—
		Mean	0·224		0 206		0·0587		0·0468

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EXPERIMENT XI. **To investigate the conditions of equilibrium and of sliding between surfaces inclined to the horizontal, as in the case of a weight placed upon an inclined plane.**

APPARATUS. The friction board, wood slider, and weights as used in Experiment V.

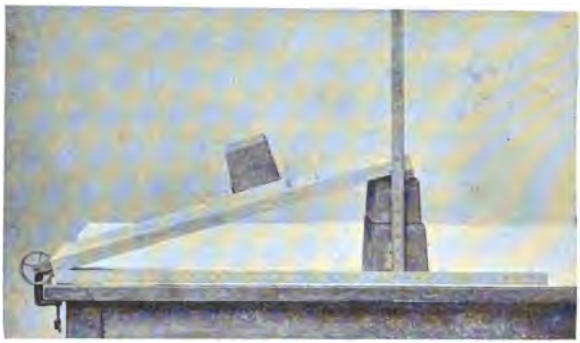

FIG. 50.

Method. (1) Place two 14 or 28 lb. weights upon the table one upon the other, and arrange the friction board, with the pulley end resting upon the table, and the other supported by the weights, so that its surface is inclined to the horizontal. Place weights on the table and against the lower end of the board, so as to keep it from slipping, or clamp the board. (See Fig. 50.) (2) Place the slider, with a weight of 7 lbs. upon it, on the upper end of the board, and then increase or decrease the angle of inclination by moving the weights in or out, until the slider moves uniformly down the board, after being started in order to overcome the friction of rest, exactly as in former experiments. (3) Find the angle of inclination by measuring the base and height as shown. (4) Repeat the experiment with weights on the slider of 14 lbs., 21 lbs., 28 lbs., and 35 lbs., measuring the angle of inclination each time.

NOTE.—The experiment may be repeated with sliders of different areas, when it should be found that if the surfaces in contact are of the same smoothness, the sliding will take place at the same angle.

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RECORD OF EXPERIMENT. *Experiment on the sliding of a loaded mahogany slider down an inclined mahogany board.*

Weight of slider = 0·78 lbs.

No.	Slider + Weights.	ANGLE OF SLIDING.		
		Base.	Perpendicular.	$\frac{\text{Perpendicular}}{\text{Base.}}$
	lbs. weight.	″	″	
1	7·781	28·45	9·30	0·325
2	14·781	28·6	9·25	0·323
3	21·781	28·50	9·25	0·324
4	28·781	28·55	9·3	0·325
5	35·781	28·4	9·75	0·343
			Mean	0·328

Angle whose $\frac{\text{perpendicular}}{\text{base}}$ or tangent = 0·328 is 18° 10′.

Further Investigation of the above Experiment. Make an outline drawing to scale of the board and slider at the angle of sliding, making the drawing as in Fig. 51. There are evidently three forces acting on the slider when it is just on the point of sliding down the plane. (1) The attraction of gravity = W acting vertically downwards. (2) The normal reaction of the plane against the slider = R, which is equal and opposite to the normal force between the slider and board. (3) The force of friction = F between the slider and board acting up the plane, and equal and opposite to the component of the attraction of gravity down the plane. Produce the direction of W upwards, and make $ab = W$. From a draw ac parallel to F; then by the triangle of forces

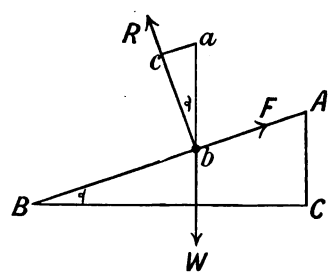

FIG. 51.

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if ab represents W, ac represents F, and bc represents R. Also the triangle abc is similar to the triangle ABC, that is, if the hypothenuse AB represents W, then the perpendicular AC represents F, and the base BC represents R.

$$\text{Now } \frac{ac}{bc}=\frac{F}{R}=\frac{\text{Friction}}{\text{Normal Force}}=\mu,$$

$$\therefore \frac{AC}{BC}=\frac{F}{R}=\mu.$$

But $\frac{AC}{BC}$ = tangent of the angle at which sliding takes place,

$\therefore$ *the tangent of the angle at which sliding takes place is equal to the coefficient of friction between the sliding surfaces.*

The angle at which sliding takes place is called the "*angle of friction*," or the "*limiting angle of friction*," or the "*angle of repose*."

RECORD OF EXPERIMENT. *To determine the angle of friction of two wood surfaces. Teak on teak, grains parallel.*

No. of Exp.	Slider and Weights.	Height.	Base.	Tan A. $\frac{\text{height}}{\text{base}}$	Value of Angle.
	lbs.	ins.	ins.		
1	2	7·40	28·65	0·2584	14°29′
2	4	6·95	28·75	0·2425	13°38′
3	6	6·625	28·85	0·2296	12°56′
4	8	6·20	28·90	0·2144	12°60′
5	10	6·40	28·90	0·2214	12°29′
6	12	6·55	28·85	0·2269	12°47′
7	14	6·25	28·92	0·2116	11°57′
8	16	6·05	29·00	0·2086	11°47′
9	18	6·05	29·00	0·2086	11°47′

This experiment is often adopted for finding the coefficient of friction.

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To determine the coefficient of friction for the same two surfaces by usual method with board horizontal.

No. of Exp.	Slider and Weights.	1st Exp. Pan and Weights.	2nd Exp. Pan and Weights.	Mean.	Mean corrected from Curve	$\mu = \frac{F}{P}$
	lbs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	lbs. ozs.	
1 18	2	0 12	0 10	0 11	0 12	0·3751
2 17	4	1 $2\frac{3}{4}$	1 $0\frac{3}{4}$	1 $1\frac{3}{4}$	1 $2\frac{3}{8}$	0·2870
3 16	6	1 10	1 $7\frac{1}{4}$	1 $8\frac{5}{8}$	1 $7\frac{3}{4}$	0·2474
4 15	8	2 $0\frac{1}{2}$	1 $14\frac{3}{4}$	1 $15\frac{5}{8}$	1 $15\frac{3}{8}$	0·2470
5 14	10	2 $8\frac{1}{2}$	2 $3\frac{1}{2}$	2 $5\frac{7}{8}$	2 $6\frac{3}{4}$	0·2422
6 13	12	2 $12\frac{3}{4}$	2 12	2 $12\frac{3}{8}$	2 12	0·2292
7 12	14	3 $2\frac{3}{4}$	3 $1\frac{1}{2}$	3 $2\frac{1}{8}$	3 $2\frac{1}{2}$	0·2255
8 11	16	3 9	3 $8\frac{1}{4}$	3 $8\frac{5}{8}$	3 $8\frac{5}{8}$	0·2208
9 10	18	3 $14\frac{3}{4}$	3 $14\frac{3}{4}$	3 $14\frac{3}{4}$	3 $14\frac{1}{4}$	0·2179

As a further illustration of what is meant by the angle of friction, let the student take a short stout stick of wood or bar of iron, and, grasping it with both hands, let him press one end against the top of a table, the stick being nearly vertical. Then, continuing to press, let him move the stick until it makes a greater and greater angle with the vertical. He will find that within a certain angle no amount of push on the rod will cause it to slip, but that immediately a certain angle is exceeded the stick slips across the table. This angle is the "angle of friction," or "angle of repose."

EXERCISES.

7. What are the laws of friction? By what experiment may they be verified?

8. State the laws of friction, and describe how you would determine the coefficient of friction between two deal boards.

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CHAPTER IX.

MACHINES.

What a Machine is. Whenever a force is applied through any distance we say that work is done. That which does the work is called an *agent*. There are many kinds of agents, as men, horses, the winds, tides, waterfalls, and machines. *A machine is therefore an agent for doing work.*[1]

Machines differ very widely in their construction, method of working, and the nature of the work they do, as, for example, we have sewing machines, lathes, cranes, and various forms of engines. We cannot conveniently experiment with such machines in an ordinary laboratory, but as they often include a number of common parts, each of which may be regarded as a machine, we are able to experiment with these. For example, a large wharf crane may be made up of chain pulley blocks, toothed wheels and worm wheels, and if we cannot experiment with the whole crane, we can do so with the separate simple machines of which it is built.

Machines in Laboratory. The machines to be described in this chapter include examples of rope and chain pulleys, the wheel and axle, screw jack, worm and wheel, halley jack and windlass. As they are all made with the same object, and behave in much the same way, it will be convenient to examine one of them in order to discover what the properties of a machine are, and how we are to investigate them.

[1] The exact definition of a machine has been given by Professor Kennedy as follows:—"A machine may be defined to be a combination of resistant bodies whose relative motions are completely constrained, and by means of which the natural energies at our disposal may be transformed into any special form of work." The student should remember this definition, and see how it fits in with what he will see in machines.

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EXPERIMENT I. **To investigate the behaviour of a set of pulley blocks when used as a machine.**

APPARATUS. (Fig. 52.) The machine will be described more fully later on, and at present we are content to note that it consists of two blocks, with three sheaves in each block, connected together with a continuous cord which starts from the upper block, and, after threading through the sheaves in both blocks, finishes with the cord to which the scale-pan is tied as shown. The lower block is provided with a hook on which weights may be hung.

The object of the machine is to enable a person pulling on the cord to which the scale-pan is tied to lift a load suspended from the lower block.

Method. (1) Hang a 28lb. weight (the *load*) on the lower block, and put sufficient weights in the scale-pan to support the load. (It is convenient to arrange the scale-pan below the load, as shown in Fig. 54.) (2) Pull the cord, so that the scale-pan moves downwards, and notice that the load is lifted. (3) Do this two or three times, and notice the distance moved by the weight and the pan during the same movement of the machine. (4) Adjust the weights in the scale-pan until it moves down uniformly after being given a start to overcome the friction of rest, and note the weights in the pan. (We may call the "pan and the weights" the *force* which lifts the load.)

What the Experiment teaches. The observations made from the above experiment ought to be :—

(*a*) That the force moves through a greater distance than the load.

(*b*) That the force is less than the load.

(*c*) That the direction of the force is changed by the action of the machine from a downward force at the scale-pan to an upward force at the hook on the lower block.

Here then we have a machine by means of which a small force applied at one part of it and acting through a long distance overcomes a much larger force at another part of it through a short distance, the direction of motion also being changed. If the student now thinks of other machines which

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he has seen at work, he should realize that these are exactly the things which they also do.

The first of these is indeed the most important property of a machine, as can be seen in any workshop by observing how a thin leather belt, moving very quickly, is able to work a powerful machine such as a planer, the tools of which move very slowly and exert considerable force. It is, perhaps, seen in the most striking manner in the use of a thin rope moving with a velocity of 1000 to 2000′ per minute for working a crane capable of lifting loads of 50 to 100 tons.

It will be seen later that all the machines to be described in this chapter are arranged to enable a small force acting through a long distance to overcome a large force through a small distance; but this is by no means the case with all machines. In the case, for example, of a steam engine driving a dynamo by means of a belt, the pressure on the piston is much greater than the tension in the belt, and the distance moved by the piston is much less than by the belt, so that here we have an instance of a large force acting through a small distance overcoming a small force through a long distance. Some machines are arranged to simply change the direction of a force.

Measurement of Work with a Machine. In order to measure the work done by a force, we multiply the magnitude of the force by the distance through which it is exerted. Our usual unit of work is the "*foot-pound*," so that we measure the force in pounds weight, and the distance in feet. By referring to the results of Experiment I., the student will see at once that the work done by the force for any movement of the machine is equal to the weight of the scale-pan and contained weights, multiplied by the distance through which they move; and that the work done upon the load is the weight of 28 lbs. multiplied by the distance through which it moves. It is very important to know what this work is with different machines, and in order to do so we must know the distance moved by the force and the load during the same movement of the machine. This is one of the first things to measure in experimenting with machines.

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Force and Load in a Machine. The force applied to a machine in order to work it is often called the *power*. It is, however, better to restrict the use of this word to the rate at which work is done, as in the expression *horse-power;* and we shall therefore speak of the force applied to a machine as the *Force* (*P*). The force which the machine exerts overcomes a resistance, which it is convenient to speak of as the *Load* (*W*). The force is frequently spoken of as the *Effort*, or "*driving force*."

In laboratory machines the *Force* and *Load* are most conveniently applied in the form of "weights" placed in a scale-pan attached to the machine, or resting directly upon the machine.

Velocity Ratio of a Machine. (*V. R.*) The velocity ratio of a machine is the ratio of the distance moved by the force to the distance moved by the load for the same movement of the machine.

$$\text{Velocity ratio} = \frac{\text{distance moved by force}}{\text{distance moved by load}}.$$

The velocity ratio of any machine is a geometric constant, and may thus be found by geometrical reasoning, if the sizes of the parts are known. These are not always easy to measure, and hence it is customary to obtain the ratio by direct measurement. Whenever possible it is better to adopt both methods, one acting as a useful check on the other.

EXPERIMENT II. **To find the velocity ratio of the following machines by experiment and by theory: (1) three-sheave cord pulley blocks, (2) wheel and axle, (3) differential chain pulley block, (4) screw jack, (5) worm and wheel, (6) halley jack, (7) windlass.**

(1) **Three-Sheave Pulley Blocks.** (Fig. 52.) *Description.* The two blocks each contain three sheaves or pulleys of equal size placed side by side upon the same axis. The cord is continuous; one end is tied to the upper block, after which it passes under one of the outside pulleys in the lower block,

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then over one in the upper block, and so on until all six pulleys have been threaded, when it hangs loose from the upper block, with a scale-pan for the force attached to the free end. The pulleys are suspended from the upper block, and the load is hung on the lower block. There may be any number of pulleys in each block, and the arrangement is known as the "second system of pulleys." It is the most common and convenient system in use.

Fig. 52.

The pulleys may be of iron or brass, about 2″ diameter, weighing with the blocks about $1\frac{3}{4}$ lbs. for the three-sheave blocks of Fig. 52. The cord is of the quality used for window-sashes, being about $\frac{3}{16}$″ diameter. The pulleys should be arranged so that the cords hang parallel, and at such a height from the floor as to allow the force to fall a distance of from 3 to 4 feet free of the load block. If the hooks on the lower blocks are large enough, the weights for the load may be hung on without requiring a scale-pan. The most convenient pans for the force are made of stout tin about 7″ diameter, with small chains, and weighing about $\frac{1}{2}$ lb.

To find Velocity Ratio by Experiment.

Method. (1) Work the machine before commencing the experiment to see that all parts are in right order. (2) Hang a weight of 14 or 28 lbs. upon the lower block to take up any slackness, and place sufficient weights in the force scale-pan to keep the load at rest in any position in which it may be placed. (3) Arrange the pan and load so that the force scale-pan is some 3′ or 4′ from the floor, and so that it can move free of the load. (4) Measure the height of some point on the load and on the force scale-pan from the floor; then move the scale-pan downwards for a convenient distance, and again measure the height of the same points from the floor. (Note.—A corner of the weight forming the load and the upper edge of the scale-pan, or a chalk mark on the weight and the pan, will be convenient points to which to measure.) (5) Take at least three sets of measurements, moving the force through different distances in order to obtain a *mean* result. (6) Enter the results in your Laboratory note-book as follows:—

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RECORD OF EXPERIMENT. *To find velocity ratio of Three Sheave pulleys.*

	First Measurement.		Second Measurement.		Third Measurement.	
	Force.	Load.	Force.	Load.	Force.	Load.
	"	"	"	"	"	"
Height from ground before movement . . .	21·5	28·7	13·2	34	17·3	31·3
Height from ground after movement . . .	5 8	31·3	7·2	35	5·7	29·4
∴ distance moved .	15·7	2·6	6	1	11·6	1·9
∴ velocity ratio = .	$\frac{15·7}{2·6}$		$\frac{6}{1}$		$\frac{11·6}{1·9}$	
	6·04		6		6·1	

Average velocity ratio = 6·05.

Velocity Ratio by Theory. There are six cords leading from the lower or load block, so that if this block be raised 1′, each of the six cords will be slackened or shortened by 1′; therefore, to take in the slack, the force must move through 6′, so that the velocity ratio is $\frac{6'}{1'} = 6$. The velocity ratio is therefore equal to the number of cords leading from the lower block; or, if there are n pulleys in each block, the velocity ratio is equal to $2n$.

(2) **Wheel and Axle.** *Description.* The machine illustrated in Fig. 53 is known as a *compound* wheel and axle owing to the axle consisting of two parts, one being larger in diameter than the other. The load is suspended from a single pulley block, one cord of which is wound round the larger axle, while the other cord winds in the opposite direction round the smaller axle as in the figure, so that while the cord is wound on one axle it is wound off the other. The force is placed in a scale-pan attached to a cord which is wound round the wheel as shown. A *simple* wheel and axle consists of a wheel and one axle only, such as would be made of the wheel and the larger axle alone.

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FIG. 53.

The wheel and axles of Fig. 53 are of wood, the wheel being 12″ diameter × 1½″ wide, and the axles 3¾″ and 5¼″ diameter, each being 5″ long. The machine should be fixed at such a height as to allow the force-pan to fall through a distance of 3 to 4 feet.

To find Velocity Ratio by Experiment.

Method. Proceed exactly as explained for the three-sheave pulley blocks, and enter the results in your Laboratory note-book in the same way.

To find Velocity Ratio by Theory.

The distance moved by the force for one revolution of the machine is equal to the circumference of the wheel, and at the same time the load cord is wound on the larger axle a length equal to its circumference, and wound off the smaller axle a length equal to its circumference. Therefore the load cord may be regarded as being shortened by the difference in the length of the circumferences of the two axles, so that the load is raised a distance of *half* this difference. Therefore the velocity ratio is the ratio of the circumference of the wheel to half the difference of the circumference of the two axles.

Let d^1 = diameter of wheel, d_2 = diameter of large axle, and d_3 diameter of small axle; then $V.R. = (d^1 \times \pi) \div \frac{1}{2}\pi(d_2 - d_3) = \frac{2d^1}{d_2 - d_3}$.

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Thickness of Cord. The above reasoning neglects the thickness of the cord, but in experimental work this must not be done, as the distance moved by the force and load with the above machine varies with the thickness of the cord used. This can easily be shown by substituting a thin cord for the thick one on the wheel, or by measuring the circumference of the wheel with a thin and thick cord. The effect of the thickness of the cord is to increase the diameter of the wheel. Let t = thickness of cord on wheel, and t_2 thickness of cord on axle; then $V.R. =$

$$(d_1+t)\pi \div \tfrac{1}{2}\pi\,\{(d_2+t_2)-(d_3+t_2)\} = \frac{2(d_1+t)}{d_2-d_3}.$$

(3) **Differential Pulley Block.** *Description.* The principle of the wheel and axle has been very usefully applied in the differential pulley block illustrated in Fig. 54. The upper block is in one piece, and forms two pulleys, one being slightly larger than the other. These two pulleys are on the same spindle, and turn together, and correspond to the compound axle of Fig. 53. The bottom pulley is a movable one, and supports the load, and an endless chain passes round the pulleys, as shown. The grooves in the upper pulley are provided with projections which fit the links of the chain. The load does not slip or run down when the force is removed, a property of the machine which makes it so exceedingly useful in practice, and the reason for which will be discovered later. A convenient size of differential pulley block for laboratory use is the $\frac{1}{4}$ ton block, where the upper pulleys are about $3\frac{1}{4}''$ and $2\frac{1}{2}''$ diameter, and the lower pulley $3''$ diameter, weighing $2\frac{3}{4}$ lbs. The chain may be of any desired length, a length of about 4 feet being required for each foot of lift.

FIG. 54.

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To find Velocity Ratio by Experiment.

Method. Proceed exactly as with the cord pulley blocks and the wheel and axle, entering your results in the same way.

Velocity Ratio by Theory. In order to raise the load the force chain must be pulled so as to wind off the larger pulley, and thus wind up the load chain on the same pulley and off the smaller one. To lower the load the force chain is pulled to wind off the small pulley and wind the load chain off the large pulley and on the small one. The rise or fall of the load in each case is therefore due to the difference in diameters of the two pulleys or grooves, so that the principle is precisely the same as in the compound wheel and axle, already described. Let d_1 = diameter of large groove, and d_2 = diameter of small groove, then velocity ratio for raising the load = $\frac{2d_1}{d_1 - d_2}$, and for lowering the load = $\frac{2d_2}{d_1 - d_2}$.

(Note that the diameter in the numerator must allow for the thickness of the chain.)

(4) **Screw Jack.** *Description.* In the machine shown in Fig. 55 the head of the screw has fixed to it a wood pulley, round which passes the cord to which the force scale-pan is attached. Above the pulley is fixed a board, on which the load is placed. The jack shown is known as a 2 ton jack, and has a screw of $\frac{1}{2}$" pitch and $5\frac{1}{2}$" long. The wood pulley is $6\frac{1}{2}$" diameter, and the small pulley for changing the direction of the cord is of brass or other metal, about $2\frac{1}{2}$" diameter, made as light and easy turning as possible.

FIG. 55.

To find Velocity Ratio by Experiment.

Method. (1) Place a 28 lb. weight upon the load board, and

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put weights in the force scale-pan to take up any slackness, but not sufficient to cause motion. Let the force scale-pan be near the top guide pulley. (2) Measure the height from the stand on which the screw jack is fitted to the upper or lower edge of the pulley or load block, and measure the height of some point on the force scale-pan from the floor as before. (3) Move the scale-pan downwards until it nearly touches the floor, and then measure again the distances for the load and force-pan from the stand and floor respectively. (NOTE.—This should be done twice, and for a third way adopt the following method.) (4) Remove the scale-pan and unwind the force cord from the pulley for a complete circumference, and measure the length of cord thus unwound. It will be equal to the distance moved by the force for one revolution of the pulley. (5) Make a chalk mark on the pulley opposite some mark or fixed point on the stand or wall, and carefully measure the height of the top or bottom edge of the pulley from the stand as before. Then turn the pulley round completely once, so that the chalk mark coincides with the fixed mark, and again measure the height of the pulley from the stand. The difference of the two measurements will be the distance moved by the load for one revolution of the pulley. (6) Record the results in your Laboratory note-book as follows:—

RECORD OF EXPERIMENT. *To find the velocity ratio of a screw jack.*

	First Measurement.		Second Measurement.	
	Force.	Load.	Force.	Load.
	"	"	"	"
Height from ground or stand before movement	29·4	10·2	24	10·3
Height from ground or stand after movement	7	10·6	8	10·7
∴ distance moved . . .	22·4	0·4	23·2	0·4
∴ velocity ratio = . . .	$\frac{22·4}{0·4}$	=56	$\frac{23·2}{0·4}$	=58

Third Measurement.

Length of force cord unwound from pulley = 22·7"
Height of load pulley before revolution = 10·25"
" " after " = 10·65"
∴ distance moved by load = 0·4"
∴ velocity ratio $= \frac{22·7}{0·4} = 56·7$
Average velocity ratio = 56·9.

Velocity Ratio by Theory. In one revolution of the screw it moves up a distance equal to its pitch, and therefore raises the load through that distance. At the same time the force cord is unwound a distance equal to the circumference of

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the wood pulley, allowing for the thickness of the cord, so that the velocity ratio is the ratio of the circumference of the wood pulley to the pitch of the screw. If d = diameter of force pulley, t = thickness of cord, and p = pitch of screw, then

$$V.R. = \frac{\pi (d+t)}{p}.$$

(5) **Worm and Wheel.** *Description.* In the machine shown in Fig. 56, the horizontal endless screw or worm is provided with a wood pulley round which the force cord is wound, and a wood drum is fixed to the worm wheel, and turns with it, carrying the cord for the load. When the force scale-pan moves downwards, the worm turns the wheel and drum, and thus raises the load. The worm shown is single-threaded with a pitch of 1″, and the worm wheel is 28″ diameter, with 90 teeth. The pulley for the force scale-pan is 6″ diameter, and the drum for the load is 12″ diameter.

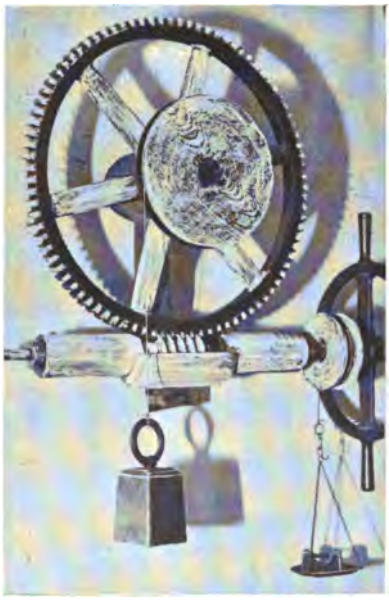

Fig. 56.

To find Velocity Ratio by Experiment.

Method. Proceed exactly as with the cord pulley blocks and other machines, by measuring the height of the force and load from the floor or other convenient place, before and after movement. Or measure the length of cord unwound from the force and load pulleys for one complete revolution, and count the number of teeth in the worm wheel. Then distance moved by force for one revolution of worm is length of cord unwound from force pulley, and distance moved by load for same movement is $\frac{\text{length of cord unwound from load pulley}}{\text{number of teeth in worm wheel}}$.

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Velocity Ratio by Theory. If the screw or worm is single threaded, it will turn the wheel through $\frac{1}{n}$th part of a revolution, where n is the number of teeth in the wheel. Therefore the worm must make n revolutions for one revolution of the wheel; and the force will move a distance of n times the circumference of its pulley, while the load is lifted a distance equal to the circumference of its drum, allowing in each case for the thickness of the cord. The velocity ratio is therefore the ratio of $n \times$ circumference of force pulley to the circumference of the load drum. If d = diameter of force pulley, t = thickness of force cord, d_2 = diameter of load drum, and t_2 = thickness of load cord, then $V.R. = \frac{n(d+t)}{d_2+t_2}$.

(6) **Halley Jack.** (Fig. 57.) *Description.* In this machine the worm and wheel are combined with a screw and nut. The square thread screw is similar to the screw of the screw jack already described, and has a board fixed to its upper end on which the load is placed. The nut for the screw is placed inside the upper end of the casing, its only movement being one of rotation. The outer rim of this nut is provided with teeth, and forms a small worm wheel, into which works a worm or screw, the spindle of which

FIG. 57.

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projects from the casing. A wood pulley for the force cord is fixed on this spindle as shown, and as the velocity ratio of the machine is very great, it is advisable to take the force cord upwards and over a small frictionless pulley suspended at some distance from the floor in order to gain enough fall for the force. The turning of the force pulley turns the worm, which causes the nut to revolve, thus making the screw move up or down. The machine shown in Fig. 57 is known as a halley jack, the screw being $1\frac{3}{4}''$ diameter and $21''$ long; the force pulley is $2\frac{1}{2}''$ diameter.

To find Velocity Ratio by Experiment.

Method. Proceed exactly as with the screw jack (machine No. 4) by measuring the distance moved by the load board and the force scale-pan before and after movement A second method is as follows: (1) Count the number of teeth in the worm wheel. (2) Measure the length of cord unwound from the force pulley in one complete revolution. (3) Measure the height of the load board above the floor. (4) Make a chalk mark on the force pulley opposite to some fixed mark on the machine, and then turn the pulley round the same number of times as there are teeth in the worm wheel. This will give one complete revolution to the worm wheel, which, acting as a nut, will cause the screw and load board to move up or down. (5) Again measure the height of the load board from the floor, and the difference between this and the measurement of (3) will be the distance moved by the load. (6) The distance moved by the force for the same movement will be the length of cord unwound from the force pulley for one complete revolution as measured in (2) *multiplied* by the number of teeth in the worm wheel.

To find Velocity Ratio by Theory. If there are n teeth in the worm wheel, the force pulley must make n revolutions for one complete revolution of the worm wheel; and as this acts as a nut, the screw will move a distance equal to its pitch. Therefore the velocity ratio is the ratio of $n \times$ circumference of force pulley to the pitch of the screw, allowing for the thickness of the force cord. If d = diameter of force pulley, t = thickness of force cord, p = pitch of screw, then $V.R. = \frac{n(d+t)\pi}{p}$.

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(7) **Windlass, or Crab-Winch.** *Description.* The object of experimenting with this machine is to discover the action and working of toothed wheels in gear, or of a "*train of wheels.*" The machine may be either "*single purchase*" or "*double purchase.*" The load is suspended by a cord from the drum shown, and for purposes of experiment the force is applied by a cord wound round the wood pulley as shown, which replaces the handle of ordinary practical use. For single purchase (Fig. 58) the large wheel on the drum axle gears directly with the small wheel, or pinion, on the axle to which the force pulley is fixed. For double purchase (Fig. 59) the large wheel on the drum axle gears with the small wheel, or pinion, on the back axle, and the large wheel on this axle gears with the small wheel, or pinion, on the force wheel axle. The machine shown is known as a 3 ton winch, and weighs in all about $2\frac{1}{2}$ cwts. The drum is 5″ diameter, and the force pulley is 16″ diameter.

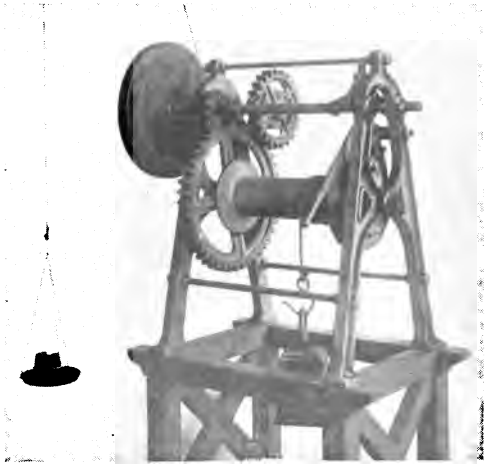

FIG. 58.

SINGLE PURCHASE.

Wheel (*A*) on drum, or load axle, in gear with pinion (*B*) on force axle. Pinion (*C*) and wheel (*D*) on back axle not in use.

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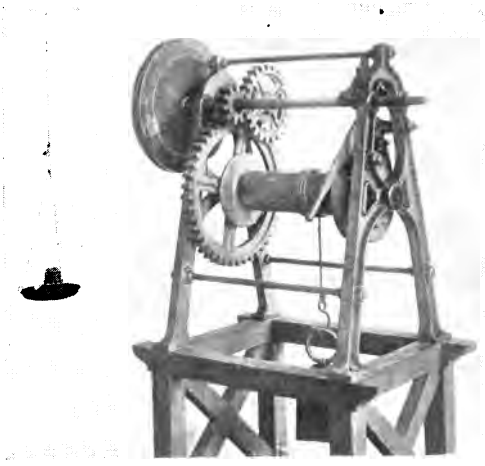

FIG. 59.

DOUBLE PURCHASE.

Wheel (A) on drum, or load axle, in gear with pinion (C) on back axle, and wheel (D) on same axle in gear with pinion (B) on force axle.

To find Velocity Ratio by Experiment.

Method. (1) Proceed exactly as with other machines by measuring the distance of the force and load from the floor before and after movement. A second method is as follows: (1) Measure the length of cord unwound from the force pulley and the load drum for one complete revolution of each. (2) Make a chalk mark on the force pulley and the load drum opposite to some fixed mark on the machine, and then revolve the force pulley until the load drum makes one complete revolution, counting the number of revolutions of the force pulley as you do so. (3) The distance moved by the load will be the length of cord unwound from the drum as measured in (1), and the distance moved by the force will be the length of cord unwound from the force pulley for one revolution, *multiplied* by the number of revolutions made by this pulley for one revolution of the drum as measured in (1) and (2).

To find Velocity Ratio by Theory. Let d = diameter of load drum, and t = thickness of load cord; d_1 = diameter of force pulley, and t_1 = thickness of force cord. Let

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number of teeth in wheel (A) on drum axle = n, in pinion (B) on force axle = n_1, in pinion (C) on back axle = n_2, and in wheel (D) on back axle = n_3, then for single purchase the force pulley must turn $\frac{n}{n_1}$ times for one revolution of drum and velocity ratio = $\frac{d_1+t_1}{d+t} \times \frac{n}{n_1}$. For double purchase the force wheel must turn $\frac{n \times n_3}{n_2 \times n_1}$ times for one revolution of drum, and velocity ratio = $\frac{d_1+t_1}{d+t} \times \frac{n \times n_3}{n_2 \times n_1}$. *

Recording Results of Experiments on Velocity Ratio of Machines. After completing the experiment with each machine, and recording the separate results, the student should enter all the results in his Laboratory note-book in the form of a table, showing the mean velocity ratio as found by experiment, and as found by theory.

RECORD OF EXPERIMENTS. *The velocity ratio of different machines.*

No.	Machine.	VELOCITY RATIO.	
		Mean of Three Results by Experiment.	By Theory.
1	Three-sheave pulley blocks .	6·04	6
2	Wheel and axle . . .	12·2	12·08
3	Differential chain pulley .	15 8	16
4	Screw jack	56·9	59·9
5	Worm and wheel . . .	45·76	45·7
6	Halley jack	216·6	225·4
7	Windlass (double purchase) .	31·5	31·05

* The student should test these statements by counting the number of teeth in the different wheels, and the revolutions of each.

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Mechanical Advantage of a Machine. It is convenient to be able to express in a simple way the advantage gained mechanically by using a machine. For example, in an experiment with a three-sheave pulley block, it was found that a force of 8·3 lbs. was required to lift a load of 28 lbs., while with a screw jack a force of 2·03 lbs. was sufficient to lift the same load. We express this by saying that the mechanical advantage of the first machine is $\frac{28}{8 \cdot 3} = 3\frac{1}{4}$, and of the second is $\frac{28}{2 \cdot 03} = 13\frac{7}{8}$, and we know from this that the load lifted with the first machine is about $3\frac{1}{4}$ times the force and in the second machine about $13\frac{7}{8}$ times the force. We have therefore the following definition :—

The mechanical advantage of a machine is the ratio of the load lifted to the force applied.

The mechanical advantage may be expressed by the letters *M A*, and, using the letters *W* and *P* for the load and force respectively, we have

$$\text{Mechanical Advantage} = \frac{\text{Load}}{\text{Force}} \text{ or } MA = \frac{W}{P}.$$

EXPERIMENT III. **To find the force required to lift loads of 14, 28, 42, and 56 lbs. with the following machines, and from the results obtained to find the Mechanical Advantage of the machines. No. 1, Three-sheave pulley blocks. No. 2, Wheel and axle. No. 3, Differential pulley block. No. 4, Screw jack. No. 5, Worm and wheel. No. 6, Halley jack. No. 7, Windlass.**

NOTE.—The method of experimenting is precisely the same for all the machines, so that, although the following description refers particularly to the "screw jack," students should have no difficulty in applying it to the other machines. It should be noted that the necessary provision for experimental work means that some of the machines are not quite the same as when used in actual practice in the workshops, as, for example, there is no load pulley or board attached to a screw jack when used in a workshop. All such additions may either be regarded as a part of the machine itself or taken as a part of the load. The weight of the force scale-pan should always be regarded as a part of the force, as it obviously helps in lifting the load. Before commencing an experiment with any

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machine, it is always well to see that it is in good working order, and that such parts as the axles of pulleys are sufficiently oiled.

Method. (1) Weigh the scale-pan to be used for the force, and attach it to the force cord. (2) Before putting any weights on the machine, find the force required to work the unloaded machine, adding weights to the force scale-pan until, after being given a start to overcome the friction of rest, the scale-pan moves down uniformly. (NOTE.—The reason for ensuring a uniform motion after starting has already been explained in connection with the experiments on Friction, Chapter VIII. The distance moved by the force should be some 3′ or 4′ with machines Nos. 1 to 5, and a greater distance with Nos. 6 and 7, where the machines have a greater velocity ratio.) (3) Repeat the experiment with weights on the load block of 14, 28, 42, and 56 lbs. (4) Record the results in your Laboratory note-book, working out the value of the Mechanical Advantage, and plot the load and force on squared paper, taking the load along the base line and the force along the ordinates.

RECORD OF EXPERIMENT. *To find the force required to lift loads of* 0, 14, 28, 42, *and* 56 *lbs. with a "screw jack," and to find the Mechanical Advantage.*

Weight of force scale-pan = 0·5 lbs.

Velocity ratio of machine as found from previous experiments = 56·9.

No.	Load = W.	Force = P.	Mechanical Advantage = $\frac{W}{P}$
	lbs. weight.	lbs. weight.	
1	0	0·547	—
2	14	1·25	11·2
3	28	2·03	13·8
4	42	2·56	16·4
5	56	3·12	17·9

After obtaining the results for each machine, a table showing all the results for purposes of comparison should be arranged as follows. The results should also be plotted on squared paper, taking a common base line, and using the same scale for the forces for each machine (see Fig. 60).

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Record of Experiment. *To find the force required to lift loads of* 0, 14, 28, 42, *and* 56 *lbs. with different machines, and to find the mechanical advantage.*

No.	W	Three-Sheave Pulley.		Wheel and Axle.		Differential pulley block.		Screw Jack.		Worm and Wheel.		Halley Jack.		Windlass.	
		P	$\frac{W}{P}$	P	$\frac{W}{P}$	P	$\frac{W}{P}$	P	$\frac{W}{P}$	P	$\frac{W}{P}$	P	$\frac{W}{P}$	P	$\frac{W}{P}$
	lbs wt.							lbs. wt.		lbs. wt.		lbs. wt.		lbs. wt.	
1	0	0·75	—	0·5	—	1·75	—	0·547	—	11	—	1·125	—	1·97	—
2	14	5·0	2·8	1·875	7·46	4·75	2·94	1·25	11·2	13	1·07	1·5	0·93	2·40	5·83
3	28	8·68	3·22	3·5	8·0	7·5	3·73	2·03	13·8	14	2·0	1·675	1·67	2·81	9·96
4	42	12·43	3·38	5·125	8·2	10·75	3·90	2·56	16·4	15·5	2·71	2·28	1·84	3·31	12·68
5	56	16·25	3·45	6·5	8·61	13	4·30	3·12	17·9	18	3·11	2·72	2·05	3·93	14·5
V.R. from former Expts.		6·04		12.2		15·8		56·9		45·76		216·6		31·05	

Curves showing how force varies with load in different machines: Scale for load $\frac{1}{10}$″ to 1 lb. Scale for force 1″ to 4 lbs.

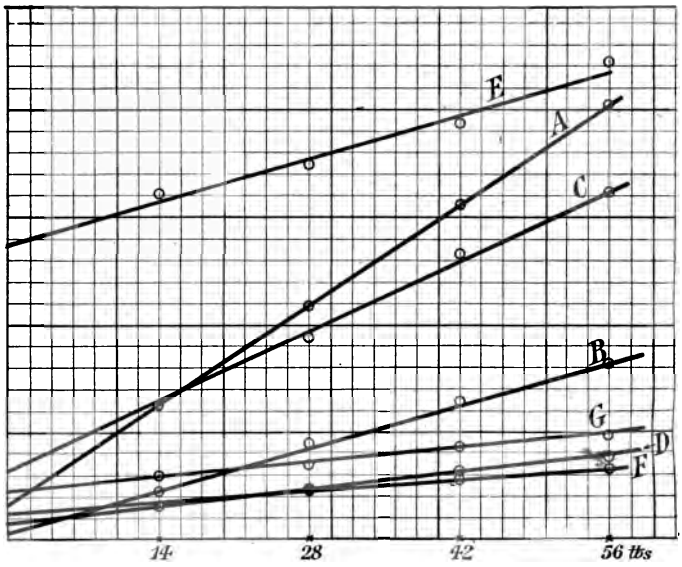

Fig. 60.

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Curve *A*—Three-sheave pulley.
„ *B*—Wheel and axle.
„ *C*—Differential pulley block.
„ *D*—Screw jack.
„ *E*—Worm and wheel.
„ *F*—Halley jack.
„ *G*—Windlass (double purchase).

The Mechanical Advantage of a Machine without Friction. If we neglect friction and the weight of the machine itself, we know from first principles that the work done by the machine in lifting the load is equal to the work put into the machine by the force. That is, if W = load, and d_1 = distance moved by load, P = force, d_2 = distance moved by force, then $P \times d_2 = W \times d_1$, or $\frac{W}{P} = \frac{d_2}{d_1}$. But $\frac{W}{P}$ is the mechanical advantage, and $\frac{d_2}{d_1}$ is the velocity ratio; therefore *without friction and other resistances due to the machine the mechanical advantage is equal to the velocity ratio.*

The mechanical advantage without friction may, however, be found directly, and by similar reasoning to that already explained for finding the velocity ratio. For example, in the system of pulleys shown in Fig. 52 there are six cords supporting the lower block. If T is the tension in the cord, the total upward force is $6T$, and neglecting friction and the machine itself, this will balance a load equal to $6T$; therefore the mechanical advantage without friction is 6. This being so, we see that the force required to lift any load, neglecting friction and the machine itself, is equal to $\frac{\text{load}}{\text{velocity ratio}}$; that is,

$$\text{Force without friction} = \frac{\text{Load}}{\text{Velocity Ratio}},$$

$$\text{or } P \text{ (without friction)} = \frac{W}{VR}.$$

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Further Considerations of the Mechanical Advantage of a Machine. By studying the table of p. 148, the student should see that the mechanical advantage of a machine is not constant, but increases with the load, and he should endeavour to understand why this is. To refer particularly to one machine only—the screw jack—he will see that the load taken for finding the mechanical advantage was simply the weights placed on the machine, and did not include the weight of the screw, pulley, and board, which have to be lifted with the load. He will see that for the first experiment a force of 0·547 lbs. was required to work the unloaded machine, that is, to lift the screw and attached parts, and to overcome the friction of the screw; and he should recognize that the force in each experiment does two things; a certain part of it is expended in working the machine, and the remainder in lifting the load. He will also see that the total load lifted by the force in any given case is equal to the weight of the screw and attached parts added to the weights on the load board. Now let him turn back to some of the experiments of Experiment III., say to that on the screw jack, and let him weigh the screw and attached parts, and add their weight to the load in each case, then divide this sum by the corresponding force, and thus obtain another ratio, which we will call the "*mechanical advantage, taking into account the weight of the machine;*" that is, if w = weight of screw and attached parts, and W = load, and P = force, as before, then the new ratio $= \frac{W+w}{P}$. The figures of this second ratio are stated in the following table for comparison with those already given in the table on p. 148, and the results are given for loads up to 112 lbs.

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Experiment with Screw Jack. Weight of screw and attached parts $= w = 8{\cdot}56$ lbs.

No.	W	$W+w$	P	$\frac{W}{P}$	$\frac{W+w}{P}$
	lbs.	lbs.	lbs. weight.		
1	0	8·56	0·547	—	—
2	14	22·56	1·25	11·2	18·04
3	28	36·56	2·03	13·8	18·01
4	42	50·56	2·56	16·4	19·80
5	56	64·56	3·12	17·9	20·70
6	70	78·56	3·81	18·4	20·60
7	84	92·56	4·37	19·2	21·2
8	98	106·56	4·94	19·8	21·6
9	112	120·56	5·56	20·1	21·7

Neglecting the results of No. 1, the student will at once notice that although the ratio $\frac{W}{P}$ increases from 11·2 to 20·1, the ratio $\frac{W+w}{P}$ for the same range of loads only increases from 18·04 to 21·7. To explain this let him plot the results on squared paper, taking the load W along the base line, and the force P along the ordinates, and let him find the equation connecting P and W, and connecting P and $W+w$, as explained on p. 73. He will find that in each case the equation is in the form of $P = a + mW$, or $P = a + m(W+w)$. Now the mechanical advantage $MA = \frac{W}{P} = \frac{W}{a+mW}$, which by dividing by $W = \frac{1}{\frac{a}{W}+m}$, from which it follows that the mechanical advantage increases with the load. If $P = mW$, then in a similar way $MA = \frac{1}{m}$, and the mechanical advantage would be a constant, whatever the load. The reason why the increase in the second ratio is less than in the first is explained by the smaller value of a in the second equation than in the first.

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The Friction of a Machine. From previous work the student will have discovered that the force required to lift any load W neglecting friction and the weight of the machine itself is equal to $\frac{W}{VR}$, and this we have already called the force without friction. He will have seen that the actual force required is always greater than the force without friction; and the question now is, How much of the actual force is used in overcoming the friction of the machine? What is it the force does in any actual experiment? It evidently does three things. (1) It lifts the load. (2) It lifts certain parts of the machine. (3) It overcomes the friction of the machine. From the principle of work, we know that the force required for the first and second of these is equal to their weight divided by the velocity ratio of the machine. The force required for No. 3 can then be found by subtraction. Thus, if P = force, W = load, w = weight of parts of machine lifted with the load, and VR = velocity ratio, then

$$P = \frac{W+w}{VR} + \text{force required to overcome friction,}$$

$$\therefore \text{ Force required to overcome friction } (F) = P - \frac{W+w}{VR},$$

$$\text{or } F = P - \frac{W+w}{VR}.$$

The force represented by F is called the *Friction of the machine.*

EXPERIMENT IV. **To find the Friction of a Differential Chain Pulley Block with loads from 0 to 112 lbs.**

Method. (1) Weigh the force scale-pan and the lower pulley block, on which the load is hung. (2) Proceed exactly as in experiments for the Mechanical Advantage by finding the force required to lift the load in each case. (3) Record the results as before, and work out the friction for each load. Plot the results on squared paper, and find the equations connecting P and W and F and W.

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RECORD OF EXPERIMENT. *To find the friction of a Differential Chain Pulley Block.*

Weight of force scale-pan = 0·5 lbs.

Weight of lower pulley block = 2·5 lbs.

Velocity ratio from previous experiments = 15·8.

No.	Load = W.	Load Producing Friction = $W+w$.	Force = P.	Force Without Friction.	Friction.
		lbs.	lbs. weight.	lbs. weight.	lbs. weight.
1	0	2·5	1·75	0·158	1·59
2	14	16·5	4·75	1·04	3·71
3	28	30·5	7·5	1·92	5·58
4	42	44·5	10·75	2·81	7·94
5	56	58·5	13	3·70	9·30
6	70	72·5	15	4·59	10·41
7	84	84·5	17·5	5·35	12·15
8	98	100·5	20·25	6·36	13·89
9	112	114·5	22·75	7·24	15·51

$P = 2\,4 + 0{\cdot}18\ W.$ $F = 2{\cdot}4 + 0{\cdot}117\ W.$

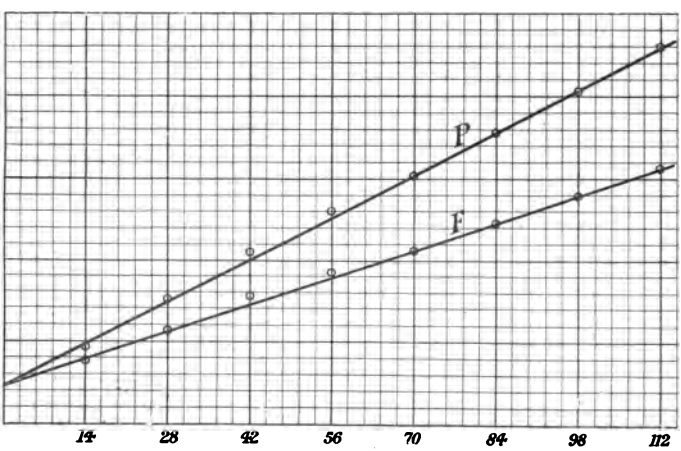

FIG. 61.

P = Force curve. F = Friction curve.

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The student can now see why it is this pulley block will not overhaul, or allow the load to run down when the force is removed. For, take the results of No. 5; the work done by the machine if the load is lifted 1 foot = $56 \times 1 = 56$ ft. lbs., and the work done by the force is $13 \times 15{\cdot}8 = 205{\cdot}4$ ft. lbs., since the VR is 15·8. Therefore, the work expended in overcoming friction for every foot of lift is $205{\cdot}4 - 56 = 149{\cdot}4$ ft. lbs., and is greater than the work which the load could do in falling. The greater the load the greater the frictional resistance; and if more than half the work which is put into the machine is used in overcoming friction, the load will not overhaul.

The Mechanical Efficiency of a Machine. From previous experiments the student will have seen that a machine cannot create energy although it transmits it, and that the energy supplied to any given machine is always lessened in transmission by the amount of such energy expended in working the machine itself. Hence it follows that the work *got out* of a machine is always less than the work *put into* it. For example, in Experiment No. 7, with the Screw Jack (p. 151), the force was 4·37 lbs. and the load 84 lbs, The velocity ratio is 56·9, so that if we assume the load to be lifted through 1 foot, the force must move through 56·9 feet. Therefore—

Work done by machine in lifting load of 84 lbs. through 1 foot = $84 \times 1 = 84$ foot lbs.

Work done by force of 4·37 lbs. weight in acting through 56·9 feet = $4{\cdot}37 \times 56{\cdot}9 = 248{\cdot}65$ foot lbs.

$\therefore$ work lost in transmission being expended in working the machine is $248{\cdot}65 - 84 = 164{\cdot}65$ foot lbs.

And the ratio of the work done by the machine to the work put into the machine = $\frac{84}{248{\cdot}65} = 0{\cdot}338$ or 33·8 per cent. This ratio is called the mechanical efficiency of the machine, and it is usually expressed as a percentage. Therefore we have the definition:—

The Mechanical Efficiency of a machine is the ratio of the work done by the machine to the work put into the machine.

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If W = load, d^1 = distance load is lifted, P = force, d^2 = distance force is exerted through, then

$$\therefore \text{ Velocity ratio} = V.R. = \frac{d^2}{d^1},$$

$$\text{Work done by machine} = W \times d^1,$$

$$\text{Work put into machine} = P \times d^2,$$

$$\text{and Mechanical Efficiency } (M.E.) = \frac{W \times d^1}{P \times d^2} = \frac{W}{P \times V.R.}$$

EXPERIMENT V. **To find the mechanical efficiency of a three-sheave pulley block with loads up to 56 lbs., and to find how the efficiency varies with the load.**

Method. (1) Proceed exactly as with former experiments by finding the force required to lift the different loads, starting with an unloaded machine, and increasing by 7 lbs. each time. (2) Work out the mechanical efficiency in each case, and plot the results on squared paper, taking the loads along the base line, and the force and mechanical efficiency along the ordinates.

RECORD OF EXPERIMENT. *To find the mechanical efficiency of a three-sheave pulley block with different loads.*

Weight of force scale-pan = 0·5 lbs.

Velocity ratio = 6.

No.	Load W	Force P	Work done by Machine.	Work put into Machine.	Mechanical Efficiency.
	lbs.	lbs. weight.	ft.-lbs.	ft.-lbs.	%
1	0	0·75	—	4·5	—
2	7	2·9	7	15·4	45·4
3	14	5·0	14	30·0	46·6
4	21	6·72	21	40·32	52·1
5	28	8·68	28	52·08	53·7
6	35	10·56	35	63·36	55·2
7	42	12·43	42	74·58	56 3
8	49	14·64	49	87·84	55·7
9	56	16·25	56	97·50	57·4

Fig. 62. P is force curve. E is mechanical efficiency curve.

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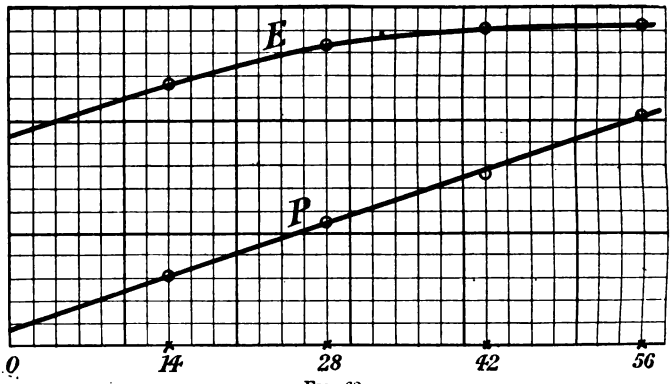

FIG. 62.

Maximum Mechanical Efficiency of a Machine. From the results of the last experiment the student will see that the mechanical efficiency increases with the load; at first the increase is somewhat rapid, and the curve rises rapidly, after which the efficiency more nearly becomes constant, as the loads increase, so as to suggest the question of whether there are conditions of maximum efficiency. Much useful work can be done in experimenting with any selected machine, say a screw jack, using much higher loads than in any experiment recorded in this chapter, and thus seeing whether a point of maximum efficiency is reached, or whether by so increasing the load as to squeeze out the oil between the threads of the screw, a point can be reached at which the efficiency actually decreases with the load. But the question can also be answered from theoretical considerations as follows: The equation connecting the force P and the load W is found in the form $P = a + mW$.

$$\text{Mechanical efficiency} = \frac{W}{P \times VR}$$

$$= \frac{W}{a + mW} \times \frac{1}{VR}$$

$$\text{(dividing by } W) = \frac{1}{\frac{a}{W} + m} \times \frac{1}{VR}.$$

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This shows that the efficiency increases with the load, since as W increases the value of $\frac{1}{\frac{a}{W}+m}$ increases, and $\frac{1}{VR}$ is a constant. If the load W becomes infinitely great, then the fraction $\frac{a}{W}$ becomes infinitely small, and may be neglected, so that with an infinitely great load the efficiency is $\frac{1}{m \times VR}$.

$$\therefore \text{ Mechanical Efficiency (maximum)} = \frac{1}{m \times VR}.$$

As m is always less than unity when VR is greater than unity, it follows that in machines where the force moves a greater distance than the load, the maximum mechanical efficiency is always greater than the inverse of the velocity ratio, that is, than $\frac{1}{VR}$.

Complete Experiment on Machines. For a complete experiment with a machine we require to find the velocity ratio, mechanical advantage, friction, and mechanical efficiency with a series of loads, which for useful and accurate results should not be less than ten in number. The first experiment for finding the force to overcome different loads should be with the machine unloaded, and the increase should be made uniformly, the last load being as great as can be conveniently managed. With machines such as the different cord pulleys and the wheel and axle, a load of 56 lbs. will probably be the maximum possible; but with the screw jack, windlass, and similar machines, there is no reason why the load should not reach to 1 cwt. or even 2 cwt. In all cases the actual experiment with the machine should be completed without any break, and results should not be worked out until the experiment is completed. The student will discover the reason for the first of these precautions if he will do part of an experiment, say, on the force required to overcome different loads with a cord pulley block, and allow a day or two to pass before completing it. After tabulating the results in the usual

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way, the student should draw curves showing the relation of the force, friction, and efficiency to the load, and should find equations connecting the force and friction with the load. The following record of a complete experiment with a halley jack will show the results to be obtained, and will serve as a guide for other machines.

RECORD OF EXPERIMENT. *Experiment to find the Mechanical Advantage, Mechanical Efficiency, and Friction of the Halley Jack.*

Weight of force-pan = 7·5 ozs.

Weight of screw and load board = 31·5 lbs.

Velocity ratio = 215·7.

No.	W	P	MA	P without F	F	ME
	lbs.	lbs.	lbs.	lbs.	lbs.	per cent.
1	0	1·16	—	—	—	—
2	28	1·80	15·5	0·27	1·53	7
3	56	2·780	20·14	·40	2·38	9·3
4	84	3·530	23·8	·53	3·0	11·2
5	112	4·405	25·4	·66	3·74	11·8
6	140	5·218	26·8	·79	4·43	12·4
7	168	6·030	27·8	·92	5·01	12·9
8	196	6·842	28·6	1·05	5·79	13·3
9	224	7·653	29·4	1·18	6·47	13·6
10	252	8·483	29·7	1·33	7·15	13·8

$$\frac{P-a}{W}=m \qquad \frac{F-a}{W}=m=\cdot 024.$$

$$m=\frac{6\cdot 03-1\cdot 08}{168}=\cdot 029 \qquad F=m\,W+a$$

$$P=\cdot 029\,W+1\cdot 08 \qquad F=\cdot 024\,W+1\cdot 08.$$

EXPERIMENT VI. **To find the mechanical advantage, friction, and mechanical efficiency of the following machines with 9 different loads. Draw curves showing the relation between the force, friction, efficiency, and the load, and find equations connecting the force and friction with the load. No. 1, cord pulley blocks (loads to 56 lbs.); No. 2, wheel and**

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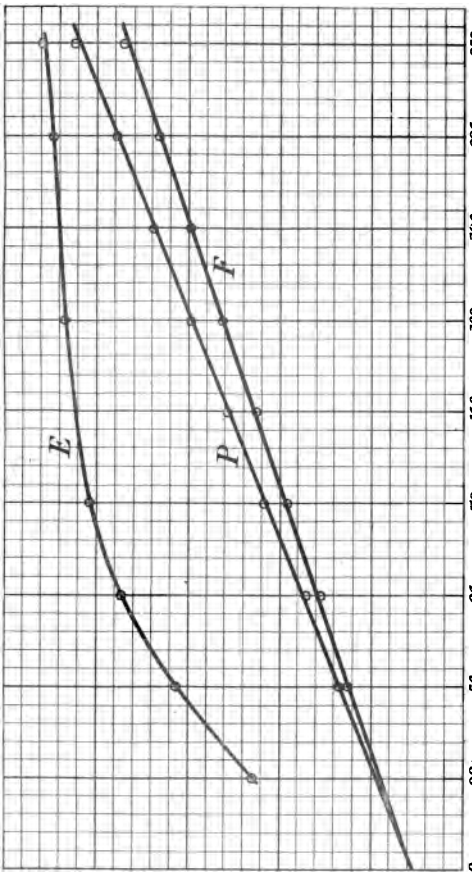
FIG. 63.

Scale: Load, 1″ = 28 lbs. Force (*P*), 1″ = 2 lbs. Friction (*F*), 1″ = 2 lbs. Mech. Eff. (*E*), 1″ = 3 %.

P is Force curve. *F* is Friction curve. *E* is Efficiency curve.



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axle (loads to 28 lbs.); No. 3, differential pulley block (loads to 112 lbs.); No. 4, screw jack (loads to 112 lbs.); No. 5, worm and wheel (loads to 112 lbs.); No. 6, halley jack (loads to 224 lbs.); No 7, windlass (loads to 224 lbs.).

The following are records of actual experiments with four different machines:—

RECORDS OF EXPERIMENTS. No. 1. *To find the mechanical advantage, friction, mechanical efficiency, velocity ratio, and maximum mechanical efficiency of single-sheaved pulley blocks with loads 8 lbs. to 57 lbs., increasing by 7 lbs.*

No. of Exp.	Load W	Force P Pan and Weights.	Mech. Adv. $\frac{\text{Load}}{\text{Force}}$	Friction (actual). $F=P-\frac{W}{VR}$	Friction (corrected) from curve.	Mechanical Efficiency.
	lbs.	lbs. ozs.		lbs. ozs.	lbs. ozs.	%
1	7	5 6½	1·48	1 6½	1 6½	74·40
2	14	9 11½	1·54	2 3½	2 3½	76·93
3	21	14 3	1·55	3 3	3 0	78·57
4	28	18 9	1·56	4 1	3 14	78·93
5	35	23 1	1·56	5 1	4 12	79·12
6	42	27 1	1·58	5 9	5 14	78·54
7	49	31 12	1·58	6 12	6 12	78·74
8	56	35 6	1·61	6 11	7 8	79·18

The velocity ratio by measurement is 2·1. Weight of load block = 1 lb.

The maximum $ME=\frac{100}{m.VR}$ where $m=\cdot6=83.\dot{3}\dot{3}\%$.

No. 2. *To find the mechanical advantage, friction, mechanical efficiency, velocity ratio, and maximum mechanical efficiency of treble-sheaved pulley blocks with loads from 7 to 56 lbs.*

Weight of load block = 1 lb. 12 ozs.

Weight of pan for force 1 lb.

Distance moved by force pan = 42·25″, while distance moved by load hook = 72″.

Therefore velocity ratio $=\frac{42\cdot25''}{7\cdot3''}=6$ nearly.

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No. of Exp.	Load W	Force P= (pan+wts.)	MA	Friction (actual) $F=P-\frac{W}{VR}$	Friction corrected from curve.	Mechanical Efficiency.
	lbs.	lbs. ozs.		lbs. ozs.	lbs. ozs.	%
1	7	2 6	2 947	1 $3\frac{1}{4}$	1 $3\frac{1}{4}$	49·11
2	14	4 8	3·222	2 $2\frac{3}{4}$	2 $0\frac{1}{4}$	53·70
3	21	6 · $7\frac{1}{2}$	3·200	3 $3\frac{1}{4}$	3 1	54·11
4	28	8 $9\frac{1}{2}$	3·368	3 15	3 $10\frac{1}{4}$	56·13
5	35	10 2	3·405	4 $4\frac{3}{4}$	4 $6\frac{1}{2}$	56·74
6	42	12 2	3·428	5 2	5 4	57·12
7	49	13 14	3·439	5 $11\frac{3}{4}$	6 $11\frac{1}{4}$	57·31
8	56	15 15	3·445	6 $9\frac{3}{4}$	6 $14\frac{3}{4}$	57·42

The velocity ratio is 6·1. Maximum $ME=\frac{100}{m.VR}$, where $m=\cdot276=60\cdot71\%$.

No. 3. *To find ME, F, MA, VR, and Max. ME for a Screw Jack.*

The weight of force pan = 1 lb.

The diameter of force pulley is $7\frac{3}{16}$". The screw is $1\frac{1}{2}$" diameter, $\frac{3}{8}$" pitch; therefore the velocity ratio is 60·1.

No. of Exp.	Load.	Force P= (pan and weights).	MA	Friction (actual) $F=P-\frac{W}{VR}$	Friction corrected from curve.	Mechanical Efficiency.
	lbs.	lbs. ozs		lbs. ozs.	lbs. ozs.	%
1	14	1 $2\frac{1}{2}$	12·39	— $14\frac{3}{4}$	— $12\frac{3}{4}$	22·39
2	28	1 $13\frac{1}{4}$	15·32	1 $4\frac{1}{3}$	1 $3\frac{1}{2}$	27 66
3	42	2 $6\frac{3}{4}$	17·34	1 $11\frac{1}{3}$	1 7	30·27
4	56	2 $15\frac{1}{2}$	18·87	2 $0\frac{2}{3}$	2 0	31·78
5	70	3 10	19·32	2 $7\frac{1}{3}$	2 $7\frac{1}{4}$	32·19
6	84	4 $3\frac{1}{4}$	19 98	2 12·85	2 $13\frac{3}{4}$	32·95
7	98	4 12	20·63	3 1 87	3 $3\frac{1}{4}$	33·72
8	112	5 6	20 84	3 8·14	3 10	33·94
9	126	6 0	21·00	3 14·4	3 $15\frac{1}{2}$	34·64
10	140	6 $8\frac{1}{2}$	21·43	4 3·17	4 $6\frac{3}{4}$	34·57
11	154	· 7 2	21·61	4 9	4 13	34·80
12	168	8 4	20·36	5 $7\frac{1}{2}$	5 $3\frac{1}{4}$	35·00
13	182	8 $9\frac{1}{2}$	21·17	5 13	5 $9\frac{1}{2}$	35·18
14	196	9 $6\frac{1}{2}$	21·82	6 $2\frac{1}{4}$	6 $0\frac{1}{4}$	35·20
15	210	10 0	21·00	6 8	6 4	35·90

The velocity ratio is 60·1. Maximum ME from equation = 36·94%.

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No. 4. *To find mechanical advantage, friction, velocity ratio, mechanical efficiency, and maximum mechanical efficiency for a double-purchase Crab or Windlass with loads 0 to 224 lbs.*

No. of Exp.	Load W	Force P	MA	Friction (actual) $F = P - \frac{W}{VR}$	Friction corrected from curve.	Mechanical Efficiency.
	lbs.	lbs. ozs.		lbs. ozs.	lbs. ozs.	%
1	0	1 $8\frac{1}{2}$	0	1 $8\frac{1}{2}$	1 9	0
2	14	2 $2\frac{1}{2}$	9·14	1 $14\frac{1}{4}$	1 $14\frac{3}{4}$	12·30
3	28	2 12	10·18	2 $3\frac{1}{2}$	2 2	19·36
4	42	3 4	12·86	2 7	2 $8\frac{1}{2}$	23·71
5	56	4 0	14·00	2 $14\frac{3}{4}$	2 $14\frac{1}{4}$	26·93
6	70	4 $10\frac{1}{4}$	15·09	3 $4\frac{3}{4}$	3 $4\frac{1}{2}$	29·20
7	84	5 $3\frac{1}{4}$	16·14	3 $9\frac{1}{2}$	3 9	31·23
8	98	5 13	16·86	3 $14\frac{3}{4}$	3 $14\frac{3}{4}$	32·42
9	112	6 $6\frac{3}{4}$	17·44	4 1	4 0	33·62
10	126	7 0	18·00	4 $9\frac{1}{4}$	4 $8\frac{3}{4}$	34·61
11	140	7 10	18·31	4 15	4 15	35·30
12	154	8 $3\frac{1}{2}$	18·74	5 3	5 3	36·03
13	168	8 $13\frac{1}{4}$	19·03	5 $9\frac{1}{2}$	5 $9\frac{1}{2}$	36·55
14	182	9 6	19·41	5 14	5 14	37·34
15	196	9 15	19·72	6 $2\frac{3}{4}$	6 $3\frac{1}{4}$	37·81
16	210	10 8	20·00	6 $7\frac{1}{2}$	6 $7\frac{1}{2}$	38·46
17	224	11 1	20·24	6 12	6 14	38·50

The velocity ratio is 52·1.

The maximum ME is $\frac{100}{m.VR}$; where $m = \cdot 338 = 45 \cdot 2\%$.

EXERCISES.

1. Draw to scale a wheel and axle by which a man sitting in a loop at the end of a rope wound round the axle can haul himself up by pulling at a rope round the wheel with a force only one-fifth of his weight. What weight is sustained by the pivots?

2. Draw a system of pulleys with a single string going twice round each of the blocks, and find the force needed to sustain a weight of 1 ton.

3. State the *Principle of Work* as applied to any machine. A windlass is used to hoist a stone weighing half a ton. If the rope coils on to a drum 3 ins. in radius, and if there is a handle at each end, 18 ins. from the axis, find (by the Principle of Work) what force each of the two handles must exert to raise the stone.

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4. A machine is contrived by means of which a weight of 3 tons, by falling 3 ft., is able to lift a weight of 168 lbs. to a height of 100 ft. Find the work done by the falling body, and what part of the work is used up in overcoming the friction of the machine.

5. Sketch in vertical section the common screw or bottle lifting jack. The lever in such a jack is single-ended, and measures 24 ins. in length. The pitch of the screw is $\frac{3}{8}$ in. What force applied at the end of the lever would be required to raise a load of 22 cwt., the effect of friction being neglected?

6. What do you understand by the efficiency of a machine, and how is it measured? In a single-purchase crab, the pinion has 12 teeth, and the wheel has 78 teeth, the diameter of the barrel being 7 ins., and the length of lever handle 14 ins. It is found that the application of a force of 15 lbs. at the end of the handle suffices to raise a weight of 280 lbs. Find the efficiency of the machine.

7. A lifting tackle is formed of two blocks, each weighing 15 lbs.; the lower block is a single movable pulley, and the upper, or fixed, block has two sheaves. The cords are vertical, and the fast end is attached to the movable block. Sketch the arrangement, and determine what pull on the cord will support 200 lbs. hung from the movable block, and also what will then be the pressure on the point of support of the upper block.

8. If the upper block of a set of pulleys and tackle has four equal sheaves, and the lower block three equal sheaves, and if a weight of 1 ton is hung on the lower block, one end of the rope being fixed to the ground and the other end free, what pull upon the free end will raise the weight, and what distance will the weight rise for every yard of increase of length in the free end? If the rope be fastened to the lower block instead of to the ground, what pull will raise the weight?

9. Describe, with a sketch, the construction of an ordinary screw jack, with a lever handle and screw. If the pitch of the screw be $\frac{3}{4}$ in., the length of the lever handle 29 ins., what load could be lifted, neglecting friction, by a force of 19 lbs. applied to the end of the lever handle?

10. Given three pulleys, each in its own block, show in diagrams two ways of putting them together into a system, and find the relation between the power and the weight in each system.

11. Sketch the screw coupling commonly used to connect together the carriages of a railway train, and explain its action. With what force would the carriages be drawn together by the exercise of a pressure of 50 lbs. applied to the ball at the end of the arm, supposing the pitch of the screw to be $\frac{3}{8}$ in., and that the centre of the ball measures 1 ft. from the axis of the screw? Friction is to be neglected.

12. A tackle consisting of an ordinary double and treble block is employed for lifting a weight of 600 lbs. attached to the double block. What force is required, neglecting friction?

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13. Sketch an ordinary bench vice. Apply the principle of work to find the gripping force obtained when a man exerts a pressure of 20 lbs. at the end of a lever 18 ins. long, the screw having four threads per inch, the length from the hinge to the screw being 18 ins., and the length from the hinge to the jaws being 24 ins.

14. Describe either a screw jack (pitch of screw $\frac{1}{2}$ in., handle 19 ins. long), or a simple winch for lifting weights up to one ton by one man. What is the mechanical advantage, neglecting friction? Describe what sort of trial you would make to find its real mechanical advantage under various loads, and what sort of results would you expect to find?

15. Describe and sketch a system of pulleys on which (neglecting the weight of the pulleys) a force of $10\frac{1}{2}$ lbs. would balance a load of 84 lbs., and show how far the force must move in order to raise the load 3 ft.

16. Given a Weston's pulley block, show how we find the velocity ratio. How would you experimentally find the real mechanical advantage? Does the mechanical advantage depend on the load which is being lifted, and if so, in which way?

17. In a system of pulleys in which there are two blocks of three pulleys each, and one string passes round them all, find the force necessary to raise 1,780 lbs. if the weight of the lower block be 20 lbs. Through what distance must the "force" act to raise the weight 20 ft. (friction being neglected)?

18. It is required to raise a block of stone weighing 7 cwt. to a height of 6 ft. from the ground. You are provided with scaffold poles, rope, and whatever pulleys you require, but the rope will not safely bear more than 150 lbs. Describe and illustrate by a sketch the arrangement you would propose, and state what would be the actual load on the rope.

19. The screw of a press has a head of 4 ins. in diameter, and a pitch of $\frac{1}{10}$ths of an inch. A lever 5 ft. long is passed through a hole in the head so as to project 1 in. on the other side. Neglecting friction, find the pressure exerted by a pull of 40 lbs. applied (*a*) 3 ins., (*b*) 12 ins from the other end of the lever.

20. Calculate the mechanical advantage of a screw, the pitch of which is $\frac{1}{2}$ an inch, when the power is applied at the end of an arm 9 in. long. Show how this calculation is modified when you take friction into account. What would you expect the actual useful mechanical advantage to be?

21. You are required to ascertain experimentally the loss of power in raising a weight by means of an ordinary screw jack. How would you proceed?

22. Find the relation between the "force" and the "load" in that system of pulleys in which the string passes round all the pulleys.

23. What do you understand by the expression "efficiency" when applied to a machine? Explain how by an experiment you could determine the "efficiency" of a pulley block tackle.

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24. A screw jack has a screw with a pitch of half an inch. The power is applied at the end of an arm 12 ins. long. What load can you lift if you apply a steady pull of 40 lbs. at the end of the arm, (*a*) assuming there is no friction, (*b*) assuming that 65 per cent. of the work you do is spent in overcoming the friction between the screw and the nut?

25. Describe any machine workable by hand for lifting weights. Give the rule for its velocity ratio. When is its velocity ratio the same as its mechanical advantage? Describe carefully how you would make tests to determine its real mechanical advantage under various loads.

26. Distinguish between the mechanical advantage and the efficiency of a machine. With a pair of chain-geared pulley blocks the hand chain has to be pulled through 26 ft. to raise the weight 1 ft., and in order to raise half a ton a pull equal to the weight of 140 lbs. has to be exerted. Find the mechanical advantage and the efficiency of the system.

27. Show how to calculate the mechanical advantage of a screw. What force must be exerted to raise 1 cwt. on a screw, the thread of which makes 20 turns in the height of 12 ins., the length of the arm of the screw being 2 ft., and the mechanical efficiency being 40 per cent.?

28. With a pair of chain-geared pulley blocks it is found that to raise the weight 1 ft. the hand chain has to be pulled through 26 ft., and that to lift 5 cwt. an average pull has to be exerted on the hand chain equal to the weight of 56 lbs. How much of the work done is wasted by friction, and what is the *efficiency* of the blocks? Sketch a horizontal section through the centre of the upper block.

29. The saddle of a lathe weighs 5 cwt., and it is moved along the bed of the lathe by a rack and pinion arrangement. What force applied at the end of a handle 10 ins. in length will be just capable of moving the saddle, supposing the pinion to have 12 teeth of $1\frac{1}{4}$ in. pitch, and the coefficient of friction between the saddle and lathe-bed to be 0·1, other friction being neglected? Sketch the arrangement.

30. In a Weston pulley block the sheaves are 9 ins. and $8\frac{1}{4}$ ins. in diameter. What weight could be lifted (neglecting friction) by a pull of 50 lbs. on the chain? Since in this block the weight remains suspended when there is no pull on the chain, what do you infer as to the limit of efficiency? How would you determine the actual efficiency of the apparatus?

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CHAPTER X.

MECHANISMS.

Cams. The name cam is given to a curved plate or groove which transmits motion by means of its curved edge. Cams are used in all kinds of machines for all sorts of purposes, and they differ very considerably in their shape. A cam may be a simple curved plate fixed on a shaft, and moving one end of a heavy lever up and down as in a common form of punching machine; or it may be a groove in a plate which works the shuttle of a sewing machine; or a groove in a circular ring fixed to a rotating shaft, and used for giving a reciprocating movement to a lever. Indeed, almost any kind of movement can be produced by means of cams.

The simplest form of cam motion is that which takes place in only one plane. Of these there are several typical or common forms with which the student ought to be familiar. One is the *Heart-shaped* cam (Fig. 64), used for moving a piece to and fro with uniform velocity; a second is the *Involute* cam (Fig. 64), used for moving a piece up gradually and allowing it to fall suddenly, as with the stamps or hammer of a machine for crushing ore; a third is the *Heart and Circle* cam (Fig. 64), used for giving a piece a uniform upward and downward movement with a period of rest; and the fourth is the simple *Eccentric* cam (Fig. 64), which consists of a circular plate fixed eccentrically upon the shaft exactly as with an eccentric sheave. For the model of Fig. 64 the cams are made of $\frac{1}{2}''$ board, and measure about $10''$ or $12'' \times 10''$; the eccentric cam being $10''$ diameter.

EXPERIMENT I. **To find the difference in the motion produced by a cam with a small and large roller, assuming the cam to rotate with uniform angular velocity.**

APPARATUS. The apparatus for experimenting with cams is shown in Fig. 64. It consists of a stout board, about $22'' \times 16''$,

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with pieces at one end to form feet so that the board can stand firmly in a vertical plane. The bar, or piece, which is to be moved by the cam, is a strip of wood about $2\frac{1}{2}$" wide moving between vertical guides. Near the lower end of the bar is a small axle for the roller, while the brass pointer shown is fixed to the bar, and moves over a scale (a strip of squared paper) on the side of the board. The lower axle shown is fastened to the board, and acts as the shaft on which the cam is fixed. Two wood rollers are provided, one 2" diameter and the other 1" diameter.

The cams are divided on one face into 12 equal parts by radial lines, and when placed on the cam axle can easily be turned round, so that any one of the lines is vertical, and coincides with the axis of the bar and roller.

FIG. 64.

Eccentric Cam.
Involute Cam. Heart Cam.
Heart and Circle Cam.

Method. (1) Fix the large roller on the roller axle, and set the cam so that the roller is in its lowest position. Read the position of the pointer on the scale. (2) Turn the cam round through $\frac{1}{12}$th of its revolution, and note the reading of the pointer on the scale. (3) Do this for each of the 12 divisions of the cam, in each case noting the reading of the pointer from its lowest starting position. (4) Repeat the experiment with the small roller. (5) Draw curves on squared paper showing the movement of the bar, by setting off 12 equal distances along the base to represent the equal intervals of time of the complete revolution of the cam, and by setting off along the ordinates the distance moved by the bar from the starting point.

NOTE.—Curves plotted in this way for the eccentric cam are shown in Fig. 66. Curve *A* 2" roller, curve *B* 1" roller.

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RECORD OF EXPERIMENT. *Movement of 2″ and 1″ diameter rollers by Eccentric Cam.*

Twelfths of Revolution of Cam.	Displacement of Roller.	
	2 inches.	1 inch.
1	·19	·13
2	·97	·85
3	2·15	1·91
4	4·00	3·84
5	5·41	5·35
6	6·00	6 00
7	5·40	5 35
8	4·01	3 84
9	2·16	1 92
10	·97	·85
11	0·24	·14
12	00	00

What this Experiment Teaches. Unless the rollers used have been nearly the same diameter, the student will find that the movement for the same part of the revolution of the cam is not identical, and that the curves drawn do not coincide.

If he looks at the contact point of the cam and roller when in their lowest position, he will see that the radial line of the cam coincides with the vertical line through the centre of the roller, and therefore that the normal through the contact point coincides with the radial line on the cam. Let him see that this is the case with each roller. Then let him turn the cam to, say, the position corresponding to the second or third part of its revolution, and again note the position with each roller. He will see that the radial line on the cam no longer coincides with the normal through the point of contact of the roller, and that the difference is *less* with the small roller than with the large one. This can also be shown as follows: Let R and R^1 be two positions of the roller, for we may suppose the cam fixed, and the roller to be moved around the cam Then the travel of the roller is the distance RF between the centres

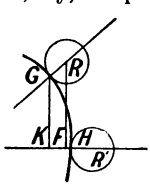

FIG. 65.

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of R and R^1. But if the rollers were the points G and H, the vertical travel would be the distance GK, which is not equal to RF. *Evidently then the movement of the bar and pointer is influenced by the diameter of the roller.*

EXPERIMENT II. **To find the movement given to a large and small roller, the axes of which move in a vertical line when in contact with different cams which rotate with uniform velocity.**

APPARATUS AND METHOD. Use the same apparatus and proceed in the same way as with Experiment I., using the two rollers with each cam. Draw curves showing the movement of the rollers for a uniform motion of the cam as in the last experiment, using a common base line, and setting off the distances moved by the rollers to the same scale in each case.

NOTE —The results for the eccentric cam are shown by the curves A and B (curve A full line, curve B dotted line) of Fig. 66.

EXPERIMENT III. **To find the velocity and acceleration of a roller moved by a cam which itself rotates with uniform angular velocity.**

APPARATUS. Use either cam and the small roller. The curves of Fig. 66 are for the Eccentric cam.

Method. Proceed exactly as in former experiments, drawing a curve on squared paper to represent the movement of the roller as before. In Fig. 66 this curve is represented by the curve A. If we regard the line ef as representing time, then the difference in the length of the ordinates $3g$ and $4h$ represent the space passed through by the roller in the interval of time represented by the line 3 4. But $\frac{\text{space}}{\text{time}}$ = average velocity,

$\therefore \frac{4h-3g}{3\ 4}$ = average velocity of the roller during the time represented by the distance 3 4. Now the part gh of this curve is nearly a straight line, so that a tangent to the curve at the point h will practically coincide with gh. Also $4h-3g=hk$, and $gk=3\ 4$, which represents the time. Therefore,

average velocity of roller $=\frac{4h-3g}{3\ 4}=\frac{hk}{gk}$, which is the tangent of the angle hgk, or of the angle made by hg with the base. The same is true for each other part of the curve between any two adjacent ordinates, assuming

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the piece of curve to be a straight line; and as the length $g\,k$ is common, the average velocity is represented by the distance $h\,k$ during the interval of time represented by $g\,k$, and by the perpendiculars of the similar triangles for other parts of the curve. Set off the distance $h\,k$ along the ordinate, midway between $3g$ and $4h$, and repeat the process for each part of the curve. Through the points thus found draw the curve C which is *the velocity curve of the roller*. (In the figure the ordinates taken from curve A are doubled in setting off curve C.) But the tangents to this curve, which is a velocity curve on a time base, will give the acceleration of the roller. Therefore treat the curve C in the same way as the curve A, and draw the curve D which is *the acceleration curve of the roller*. A fuller explanation of this is given in Chapter V., page 74. Note that the velocity curve C cuts the base line immediately under the apex of the displacement curve A, when the velocity is evidently nothing; and that the acceleration curve D cuts the base line immediately under or over the apices of the velocity curve C.

In Fig. 66 the acceleration curve D is drawn for one-half of the movement only.

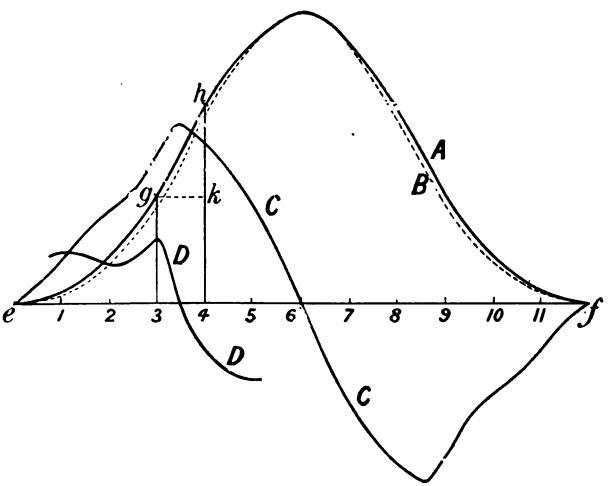

FIG. 66.

EXPERIMENT IV. **To construct a cam to give a desired rectilinear movement to a roller of 1″ diameter when the cam is fixed to a shaft which rotates with uniform angular velocity. Diameter of shaft 3″. Least metal around shaft**

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1″. Cam to raise the roller at a uniform rate 2″ in $\frac{2}{5}$ revolution, and lower it the same distance in $\frac{1}{5}$ revolution. The roller to remain at rest in the upper position for $\frac{1}{10}$th revolution, and in lower position for rest of revolution.

Method. (Fig. 67.) Take a piece of stout millboard about 8″ square. Draw a circle of radius equal to radius of shaft + radius of roller + least metal around shaft ($1\frac{1}{2}$″ + $\frac{1}{2}$ + 1″ = 3″). Divide this circle into ten equal parts by radial lines. Then mark points on each line, as shown in the figure, and draw a continuous curve through the points. This curve is the shape for a cam which will give the desired motion to a *point* in contact with it; but what we require is that the motion shall be given to a roller of 1″ diameter. To obtain this we draw another curve parallel to the first one, and $\frac{1}{2}$″ within it. This is best done by drawing a number of small arcs of $\frac{1}{2}$″ radius within the curve, using the curve as centre, and then drawing a second curve touching these arcs. This curve is shown in the figure by the dark line Cut away the cardboard to this line, finishing the edge smoothly with a file or sand-paper. Test the cam by fixing it on the cam-board as used in Experiment I., and draw a curve showing the movement of the roller as before. Find also the movement given by the cam when the large roller is used, and draw the curve for this on the same base line.

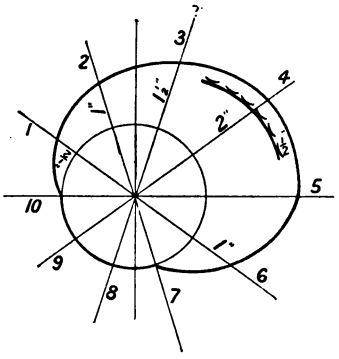

FIG. 67.

EXERCISE.

1. Sketch a cam which would give a slow forward and quick return motion to a reciprocating piece, with an interval of rest between the two motions.

Mechanisms for Reciprocating and Circular Motion. The most common example of the conversion of reciprocating motion into circular motion is that of a steam engine, where the to and fro movement of the piston in a straight line is changed to the circular motion of the crank, the mechanism by which this is effected being familiar as the crank and connecting rod. Another common example is

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in the use of an eccentric for changing the circular motion of the crank shaft into the reciprocating motion of the slide valve. A third arrangement is sometimes used in small vertical engines and pumps where the piston and pump-rod are in one piece, with a slotted bar, in which the slot is at right angles to the rod, and the crank pin moves to and fro in the slot. (See Fig. 69.) For our purpose it does not matter whether we consider the mechanism as changing reciprocating to circular motion or the reverse, as either is equally possible; but what we do want to know is what effect has the length of the connecting rod on the motion of the piston, what is the nature of the motion produced by an eccentric, and by the use of the slotted link, as in Fig. 69, instead of an ordinary connecting rod. This we can do quite easily by experiment.

FIG. 68.

EXPERIMENT V. **To investigate the motion of a connecting rod and crank, and to find the effect of using connecting rods of different lengths. The motion of the crank is supposed uniform.**

APPARATUS. (Figs. 68 and 69.) A stout board 4′-3″ × 16″ is fitted with feet so as to stand in a vertical plane. A circular piece of wood $9\frac{1}{2}$″ diameter is fixed to turn easily about its centre, and represents the crank disc. The face of the disc is divided by radial lines into twelve equal parts. The crank is represented by the wood piece shown fixed to the disc, to which one end of the connecting rod is attached. A small block of wood, which represents the crosshead of the engine, is fixed between

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two wooden guide bars, so as to move freely to and fro, the block being provided with a metal pin, to which the other end of the connecting rod can be fixed. The connecting rods simply consist of stout laths of different lengths, with a hole at each end for attaching to the crank pin and crosshead, or one rod with holes in different positions, the length of the rods being a certain number of times the length of the crank arm, say rods of 2, 3, 4, 5, and 6 times the crank. Pointers are fixed as shown for marking the position of the crank disc, and for measuring the position of the crosshead. The student should note that it is not necessary to provide a piston-rod or piston, since the motion of the crosshead is the same as that of the piston.

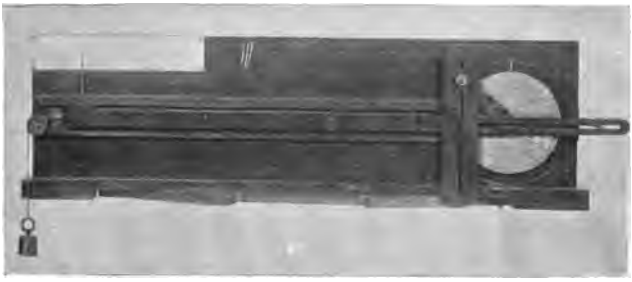

FIG. 69.

Method. (1) Using the long connecting rod (length = 6 cranks), move the crosshead to the extreme left-hand position, so that the crank arm and connecting rod are in the same straight line, and note the reading of the crosshead pointer. (2) Move the crank disc through each twelfth of the revolution for one complete revolution, and read the distance moved by the crosshead from the starting-point for each movement. (3) Repeat the experiment for a short connecting rod of 3 cranks length. (4) Repeat the experiment for the rod with the slotted link. (5) Record the results, and draw the space, velocity, and acceleration curves for each rod, as in Fig. 71, supposing the crank to move uniformly, and the time of a complete revolution to be 12 seconds. (6) Draw a skeleton diagram, as in Fig. 70, to show positions of piston at end of each second.

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Records of Experiment. *The following are the results of an experiment as just described:—Distance in inches moved by piston from position of back "dead centre" for equal movements of crank.*

	Connecting Rod of 6 Cranks.	Connecting Rod of 3 Cranks.	Slotted Link.
1st twelfth of revolution of crank	0·90	1·02	0·84
2nd " " "	3·36	3·84	3·00
3rd " " "	6·48	6·96	6·00
4th " " "	9·36	9·72	9·00
5th " " "	11·28	11·40	11·26
6th " " "	12·0	12·00	12·00
7th " " "	11·28	11·40	11·26
8th " " "	9·36	9·72	9·00
9th " " "	6·48	6·96	6·00
10th " " "	3·36	3·84	3·00
11th " " "	0·90	1·02	0·84
12th " " "	0	0	0

Distances moved by piston for uniform motion of crank.

a 6 cranks. *b* 3 cranks.
c Slotted link.

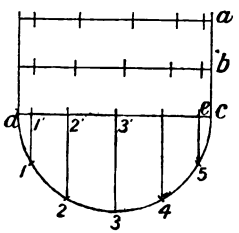

Fig. 70.

What this Experiment Teaches. The curves shown in Fig. 71 are the displacement or space curve, the velocity and the acceleration curves for the slotted link. Curve *A* is the displacement curve drawn by setting off distances to represent 1, 2, 3, 4, etc., seconds along the base, and marking along the ordinates distances equal to the distance moved by the piston from the starting-point. Curve *B* is the velocity curve, drawn by setting up midway between the seconds ordinates a distance equal to the space moved during the second. Curve *C* is part of the acceleration curve drawn by setting up midway between the velocity ordinates a distance equal to the difference in velocity during the second. (For further details of these curves see Chapter V., page 74.)

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From these curves the student will see that, assuming a uniform velocity of the crank pin, the velocity of the piston during any stroke increases from the beginning towards the middle of the stroke and then decreases towards the end of the stroke, and that the acceleration for the same period decreases from a maximum at the beginning and end of the stroke, to nothing at the middle of the stroke. The student should study these curves slowly, and be quite clear as to the relations between the displacement, velocity, and acceleration. He should then draw similar and complete curves for the connecting rod of three cranks length and of six cranks length, noting how these differ from the curves for the slotted link, and reasoning as to which is the best rod to use. As further evidence in the same direction, let him now examine the movement with the slotted link. The semi-circle drawn under Fig. 70 represents the travel of the

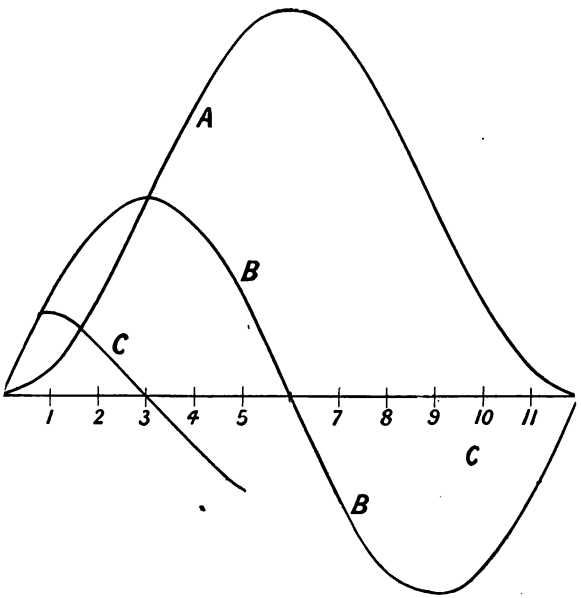

FIG. 71.

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crank pin for one stroke of the piston, and the divisions 1, 2, 3 are the equal divisions for the position of the crank pin corresponding to the position of the piston shown on the three lines *a*, *b*, *c*. Notice that the points on the line *c* exactly correspond to the points on the diameter *d*, *e*, found by drawing perpendicular lines from the divisions on the semi-circle. Now, the motion of the piston produced by the slotted link is the same as would be produced by a connecting rod moving parallel to the piston; that is a connecting rod of *infinite length*. The slotted link is therefore equivalent to a *connecting rod of infinite length*.

The motion represented by the skeleton diagram *c* Fig. 70 is known as *Simple Harmonic motion;* that is, if a body moves along the line *d e* so that it takes as long to move from *d* to 1′ as from 1′ to 2′, or from 2′ to 3′, or from any one point to the next point, its motion is said to be a *simple harmonic motion*. Therefore the motion of a piston with a connecting rod infinitely long is simple harmonic.

The student will see from the diagram of Fig. 70, as indeed from first principles, that the longer the connecting rod the more nearly does the motion of the piston approach to simple harmonic motion; also, from Fig. 71, that the shorter the connecting rod the more irregular the velocity and acceleration of the piston. He will therefore be able to reason as to why it is better to use a long rod.

EXERCISE.

2. In a direct acting engine the crank is 2′ in length, and the connecting rod is 8′. Find the distance in inches of the piston from the middle point of its stroke when the crank is at 90° from a dead centre.

Trains of Wheels. When two or more wheels are employed in mechanism it is usual to speak of them as a train of wheels. Thus in Fig. 59 the four wheels of the windlass would be spoken of as a train of wheels. The small wheel on the front axle is a *driver*, and the large wheel on the back axle, into which it gears, a *follower*. The small wheel on the same spindle is a *driver*, and the large wheel on the drum axle a *follower*.

The object in arranging a train of wheels is to obtain a

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certain number of revolutions for the last *follower*, with a fixed number of revolutions for the first *driver;* or, in other words, to ensure a certain number of revolutions for the last *follower* for one revolution of the first *driver*. The number which expresses this is known as the *value of the train of wheels*. Thus if a driver of 100 teeth drives a follower of 50 teeth, then the value of the train is $\frac{100}{50} = 2$, and the follower goes two revolutions for every single revolution of the driver. In the train of wheels of the windlass (Fig. 59) the value of the train is $\frac{10 \times 10}{23 \times 45} = \frac{100}{1035}$, or ·096.

$$\therefore \text{ Value of train} = \frac{\text{Drivers}}{\text{Followers}}.$$

In the case of belting and pulley the circumference, diameter or radius of the pulleys is used in finding this ratio; and in the case of toothed wheels the number of teeth, or the radius or diameter of the pitch circles.

One of the most common examples of the arrangement of a train of wheels is in a screw cutting lathe, where the wheels are used for driving the leading screw of the lathe from the mandril. An ordinary arrangement, as found on almost any lathe, is shown in Fig 72. The lathe mandril drives the small upper wheel or pinion *A*, and the leading screw is the bottom right-hand spindle *B*. The bracket shown is fixed to turn about this spindle as a centre, and is arranged to carry wheels for completing the train. The large upper wheel *D* has a small pinion *C* behind it, and on the same axis. The pitch of the

Fig. 72.

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leading screw or the number of threads per inch is known, and the object of the train is so to regulate the revolutions of the leading screw, compared with the revolutions of the mandril, as to be able to cut a thread of a required pitch or number of threads per inch on a piece which turns with the mandril. Thus suppose the leading screw to have 4 threads per inch, and we require to cut a screw of 12 threads per inch, then the value of the train, calling the driver A and the follower B, must be $\frac{4}{12}=\frac{1}{3}$; that is, $\frac{A\times C}{D\times B}=\frac{1}{3}$. Therefore the wheels may be $\frac{10\times 60}{30\times 60}$, or more conveniently $=\frac{20\times 60}{90\times 40}$, or any four wheels, so that value of $\frac{\text{drivers}}{\text{followers}}=\frac{1}{3}$.

EXPERIMENTS. **Make the following experiments with trains of wheels, using the change wheels and screw cutting arrangement of an ordinary lathe.**

EXPERIMENT VI. **Fix a 20 wheel on the mandril and a 40 wheel on the leading screw, and find in what way any single intermediate wheel working between and with the two affects the motion of the 40 wheel relatively to the 20 wheel. Note both the rate of motion and the direction of motion.**

EXPERIMENT VII. **Arrange a train of wheels for cutting a screw of 16 threads per inch. Test your work by placing a pencil in the slide rest, and a smooth wood drum of about 1″ diameter in the lathe centres, and allowing the pencil to trace the thread upon the drum.**

EXPERIMENT VIII. **Arrange a train of wheels so that the leading screw may make $\frac{1}{80}$ revolution for one revolution of the mandril. Test by counting.**

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EXERCISES.

3. Three spur wheels—*A*, *B*, *C*—are on parallel axes, and are in gear. *A* has 10 teeth, *B* has 35 teeth, and *C* has 55 teeth. How many revolutions upon its axis will be made by *A* for every four revolutions of *C*? Why is *B* called an *idle* wheel, and what is its use?

4. Explain the manner in which change wheels are employed in cutting screw threads of different pitches in the same lathe. What number of wheels would you employ, how would they be arranged, and how many teeth would each wheel have in order to cut upon a bolt a left-handed screw thread of $\frac{3}{8}$ in. pitch, on a lathe whose leading screw is right-handed, and has two threads to the inch?

5. If the slide rest of a lathe traverses the bed at the rate of $1\frac{1}{8}$ ft. when the leading screw makes 56 revolutions, what is the pitch of the screw thread?

6. Why is a lathe often back-geared? If the two wheels have 63 and 63 teeth respectively, and each pinion has 25 teeth, find the reduction in the velocity ratio of the lathe spindle due to the back-gear.

7. You are required to cut a left-handed screw of five threads to the inch in a lathe fitted with a right-handed guide screw of $\frac{1}{2}$ in. pitch. Show clearly by the aid of sketches the change wheels which you would employ for the purpose, indicating how they would be respectively carried, and the number of teeth in each wheel.

8. Given a screw-cutting lathe with a right-handed leading screw with four threads per inch, sketch an arrangement for cutting a left-hand thread of eleven threads per inch. What gear wheels would be required?

9. Sketch and describe the mechanism for cutting a screw with five threads to the inch in a lathe where the guide screw has three threads to the inch. Assign suitable numbers to the wheels which you would employ.

Velocity Ratio of Shafting and Slip of Belts. The very common method of driving shafting and machines by means of belting and pulleys will be familiar to all students. They will know that the reason one pulley can drive another when connected by a belt is due to the friction between the belt and the pulleys. Nor can they have failed to notice that when the machine being driven is required at some particular time to do more work than usual, the driving belt often *slips* on the pulley, and continues to move without turning the pulley. In working a lathe, it is by no means unusual to find the belt slip if the tool is set to take a specially heavy cut.

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If we have a shaft *A* revolving 120 revolutions per minute, and we wish to drive another shaft *B* from it by means of pulleys and a belt, so that it shall revolve at twice the speed, we know that we have only to choose the pulleys so that the pulley on shaft *A* shall be twice the diameter of the pulley on shaft *B*. But supposing the belt slips on either pulley, then it is clearly by no means certain that the shaft *B* will revolve at twice the speed of shaft *A*. This means a change in the velocity ratio of the shafting, which may be a bad thing with certain machines. It also means *wasted work*, exactly as when the wheels of a locomotive skid on the slippery rails, and this is, perhaps, more serious than change of speed. It is therefore interesting and useful to know something of the conditions under which belts slip and the extent of the slip, and as most teaching institutes contain a workshop of some kind where shafting is fitted, the following experiments are introduced as affording opportunity for acquiring valuable knowledge.

It is convenient to express the slip of belting as a percentage. Thus, supposing we are finding the slip between two shafts *A* and *B*, and we find that whereas *B* ought to be making 200 revolutions per minute, it actually makes only 190 revolutions, then the slip is $200 - 190 = 10$ revolutions, or $\frac{10 \times 100}{200} = 5\ \%$.

EXPERIMENT IX. **To determine the slip between the belts and pulleys of shafting, and the velocity ratio of the shafting.**

APPARATUS. A usual arrangement in a workshop is to find a machine driven from a pulley on a counter shaft, while the counter shaft is driven by a belt on another pulley to a pulley on the main driving shaft. This means two pairs of pulleys and two belts. Very frequently there will be another belt from the engine to the driving shaft. The slip of any single belt, or the slip between the engine and machine, including all three belts, can be determined. The following explanation assumes the condition of two belts and four pulleys, as in the case of a machine driven from the main shaft through a counter shaft, but the student will understand that the method is applicable to any arrangement.

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Method. (1) Measure the circumference of each pulley with a piece of thin flexible string. If the rim of the pulley is rounded it is better to measure the largest circumference. (Callipers may be used with very small pulleys.) (2) Set the shafting in motion, and station observers to count the number of revolutions of the main shaft and the driving shaft during the same interval of time, say in 1 or 2 minutes. This may be done quite easily by chalking a mark on the pulleys. (3) Take similar observations for the driving pulley on the counter shaft and the pulley on the machine, and again between the pulley on the main driving shaft and the pulley on the machine, thus including both belts. (4) Calculate from the sizes taken what should be the speed of the counter shaft and of the machine, and then work out the slip for each belt separately, and for the pair of belts. (5) Express the slip as a percentage, calculate the actual velocity ratios of the shafting, and make a sketch of the pulleys, and enter the results of your experiments in your Laboratory note-book.

The following are the results of an experiment similar to the one described :—

Circumference of driver pulley on main shaft $= 25{\cdot}5''$
,, follower ,, counter shaft $= 29{\cdot}5''$
,, driver ,, ,, $= 26{\cdot}25''$
,, follower ,, machine $= 33{\cdot}125''$

NUMBER OF REVOLUTIONS OF PULLEYS IN ONE MINUTE.

No. 1 { driver pulley on main shaft . . $= 109\frac{1}{4}$
follower ,, counter shaft . . $= 92\frac{1}{2}$

No. 2 { driver pulley on counter shaft . . $= 92\frac{1}{2}$
follower ,, machine . . $= 70\frac{5}{8}$

No. 3 { driver pulley on main shaft . . $= 109\frac{1}{4}$
follower ,, machine . . $= 70\frac{5}{8}$

No. 1 { Revolutions of counter shaft without slip $= \frac{25{\cdot}5 \times 109{\cdot}5}{29{\cdot}5} = 94{\cdot}66$
,, ,, ,, actual . $= 92\frac{1}{2}$ revolutions
$\therefore$ slip $= 2{\cdot}16$ revolutions $= \frac{2{\cdot}16 \times 100}{94{\cdot}66} = 2{\cdot}28\%$

No. 2 { Revolutions of machine without slip . $= \frac{92{\cdot}5 \times 26\ 25}{33{\cdot}125} = 73{\cdot}3$
,, ,, actual . . $= 70\frac{5}{8}$
$\therefore$ slip $= 2\ 67$ revolutions $= \frac{2{\cdot}67 \times 100}{73{\cdot}3} = 3{\cdot}65\%$

No. 3 { Revolutions of machine without slip $= \frac{26{\cdot}25 \times 25{\cdot}5 \times 109{\cdot}25}{29{\cdot}5 \times 33{\cdot}125} = 75{\cdot}02$[1]
,, ,, actual . $= 70\frac{5}{8}$
$\therefore$ slip $= 4{\cdot}4$ revolutions $= \frac{4{\cdot}4 \times 100}{75{\cdot}02} = 5{\cdot}86\%$

[1] Note that $\frac{26{\cdot}25 \times 25{\cdot}5}{29{\cdot}5 \times 33{\cdot}125}$ represents the value of the train of the four pulleys.

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No. 1. Actual velocity ratio of main shaft and counter shaft $= \frac{109 \cdot 25}{92 \cdot 5} = 1 \cdot 18$

No. 2. ,, ,, counter shaft and machine $= \frac{92 \cdot 5}{70 \cdot 6} = 1 \cdot 31$

No. 3. ,, ,, main shaft and machine $= \frac{109 \cdot 25}{70 \cdot 6} = 1 \cdot 55$

EXPERIMENT X. **If Experiment IX. was performed with the shafting running light, that is, without the machine doing work, it should be repeated with the machine at work.**

EXERCISES.

10. Sketch an arrangement of four pulleys with bands for driving a fan at 1500 revolutions per minute from a shaft making 100 revolutions per minute.

11. A shaft making 100 revolutions per minute carries a driving pulley $2\frac{1}{2}$ ft. in diameter, and communicates motion by means of a belt to a parallel shaft at a distance of 6 ft., carrying a pulley 1 ft. in diameter. Find the speed of the belt. Find also the number of revolutions per minute of the driven shaft, allowing a slip of $1\frac{1}{2}$ per cent.

12. A main shaft carrying a pulley of 12 inches diameter, and running at 60 revolutions per minute, communicates motion by a belt to a pulley of 12 inches diameter fixed to a counter shaft. A second pulley on the counter shaft of $8\frac{3}{4}$ inches in diameter carries on the motion to a revolving spindle, which is keyed to a pulley of $4\frac{1}{4}$ inches diameter. Find the number of revolutions per minute made by this last pulley.

13. A driving shaft runs at 100 revolutions per minute, and carries a pulley 22 ins. in diameter, from which a belt communicates motion to a pulley 12 ins. in diameter carried upon a counter shaft. On the counter shaft is also a cone pulley having steps 8, 6, and 4 ins. in diameter respectively, which gives motion to another cone pulley with corresponding steps on a lathe spindle. Sketch the arrangement in front and end elevation, and find the greatest and least speeds at which the lathe spindle can revolve.

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CHAPTER XI.

HYDRAULICS. FLOW OF WATER THROUGH ORIFICES AND PIPES.

Nature of Experiments. The object of the experiments in this chapter is to enable the student to obtain some useful notions about the flow of water through orifices and pipes; a branch of hydraulics which is of great practical importance. He is supposed to know something of the science of hydrostatics, and to know for example what is meant by speaking of a "head" of water, and how the pressure on any plane below the surface of water is influenced by the "head" above the plane.

Apparatus Required for Experiments. As all the experiments to be performed require practically the same fittings and apparatus, these may be well described now. But although for convenience of description the fittings described are those in the author's laboratory, it should be clearly understood that they may differ considerably in size and arrangement, and in minor details.

Fig. 73. The upper tank is the "supply" tank, and measures 23″ deep, 26″ wide, 18″ broad, holding 40 gallons. It is fixed with its bottom about 5′ from the floor. The lower tank is the "measuring" tank, and is 20″ diameter, and 32″ deep, being fixed on wood bearers a sufficient distance above the floor to allow of a waste pipe being brought away from beneath it. Both tanks are of galvanized iron. Each is fitted with a glass water gauge made by fitting a vertical piece of glass tube into a simple bend which is fixed in one side of the tank near the bottom. A scale is fixed by the side of the tube, and is marked to show divisions of inches for the supply tank, and of "pounds" or "cubic feet" for the measuring tank; the graduation being

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done in the latter case by the direct measuring in of water. The screw valve shown near the scale of the supply tank regulates the supply of water to the tank, and is so placed that the experimenter may regulate the water supply and read the scale at the same time. The water enters the tank near the top corner, furthest away from the scale, and a vertical baffle plate or screen is fixed at about the centre of the tank parallel to the side to which the scale is fixed, and passing from top to bottom right across the tank, practically dividing it into two compartments. This screen is made of a light wood frame covered with wire gauze, and its object is to still the water so that the surface above the orifice shall be smooth and free from waves. Various other arrangements to effect the same purpose are used, one being to cover the mouth of the supply pipe with a loose bag of coarse cloth material through which the water can easily pass. In the tank described, the orifice is in the bottom plate near to the side on which the scale is fixed, but it is more usual, and perhaps better, to have the orifice in the *front* plate of the tank with its plane vertical. As orifices of different sizes and shapes are required, it is usual to cut these in circular brass plates (say about 4″ diameter, and $\frac{1}{2}$″ thick), which can be screwed on to the tank with three or four bolts and nuts as required, a thin sheet of indiarubber being inserted between the tank and plate to prevent leaking. The pipe shown from below the orifice to the measuring tank is so fixed

FIG. 73.

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as to be turned easily, so that the water may pass either into the measuring tank or into the waste pipe shown on the right-hand side of this tank and leading direct to the drain. The waste pipe from the measuring tank is of 2″ diameter, and is fixed to the bottom of the tank so that the tank may be completely emptied. An ordinary tap or cock is fitted in the pipe near the tank.

Orifices. A very large number of these may, of course, be supplied, but the following will probably be found sufficient:—

(1) Round hole $\frac{3}{8}$″ diameter with bevelled edge.
(2) ,, ,, $\frac{3}{4}$″ ,, ,, ,,
(3) ,, ,, $\frac{3}{8}$″ ,, ordinary ,,
(4) Square ,, $\frac{1}{2}$″ side with bevelled edge.
(5) Triangular ,, $\frac{1}{2}$″ ,, ,,
(6) Parallel pipe $\frac{1}{2}$″ diameter 3″ long.
(7) Increasing pipe from $\frac{1}{2}$″ to 1″ diameter 3″ long.
(8) Decreasing pipe from 1″ to $\frac{1}{2}$″ ,, 3″ ,,

EXPERIMENT I. **To determine how the quantity of water flowing through an orifice depends upon the head.**

NOTE.—Any one of the orifices Nos. 1 to 5 may be used, but it is better to use Nos. 1 or 2.

Method. (1) The quantity of water flowing through under four or six different heads should be measured; these may be 4″, 8″, 12″, 16″, 20″ and 22″. Commence with the greatest head. (2) Fill the tank, turn the delivery pipe into the waste pipe, and regulate the water supply until the head is constant at 22″. While this is being done the measuring tank should be emptied. (3) Quickly turn the delivery pipe into the measuring tank, noting the *time* at which this is done. (4) When the measuring tank is nearly full, and preferably at the end of a certain number of minutes, turn the delivery pipe into the waste pipe, and stop the supply to the supply tank. (5) Note the time of flow, the head, and the quantity of the flow. (6) Repeat the experiment with the other heads. (7) Enter the results in your note-book, and plot them on squared paper, taking "head" along the base line and "quantity" along the ordinates. State your conclusion as to how the quantity varies with the head.

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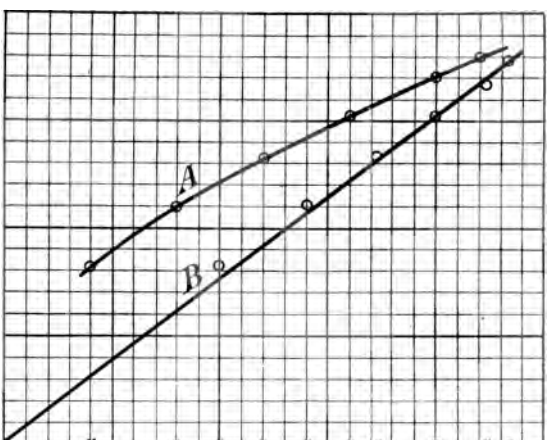
RECORD OF EXPERIMENT. *Discharge through small circular orifice 0·385″ diameter, bevelled edge. Time of flow for each experiment ten minutes.*

No.	Head H	Actual Discharge Q	Square Root Head $\sqrt{H''}$
	″	cu. ft.	
1	22	3·59	4·690
2	20	3·39	4·472
3	16	3·06	4 000
4	12	2·68	3·464
5	8	2·22	2·828
6	4	1·65	2·000



Scale for A $\frac{1}{2}$″ *to* 1″
″ ″ *B* 1″ *to* 1″

FIG. 74.

What this Experiment teaches. The above result and the curve drawn (Fig. 74 *A*) as suggested will show that the quantity is certainly not directly proportional to the head.

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For example, the flow at 16″ is 3·06 cubic feet, and at 4″ is 1·65 cubic feet. That is, when the head is increased four times, the flow is only increased 1·85 times. This should suggest that the flow might perhaps be proportional to the square root of the head—that it should have been doubled in the case just considered, since $\sqrt{4}=2$. Plot then a second curve, setting off distances equal to square root of head along the base line, and cubic feet of flow along the ordinates. This is shown in Fig. 74 *B*, and where the points found lie sufficiently near to the straight line to justify a straight line being drawn through them, thus showing that *the quantity of the flow is directly proportional to the square root of the head.* Other reasons for this will be found in the following considerations.

Velocity and Quantity of Flow. The question now arises, Can we determine in any other way than by experiment how much water will flow through a given orifice in a given time if we know the head? It is one thing to say that with a certain sharp-edged orifice of 0·385″ diameter 19·125 gallons of water will flow through in one minute under a head of 16″, and another thing to find some rule or law which enables us to calculate how much water will flow through in a certain time if we know the orifice and head. We are accustomed to say that if water is flowing through a pipe of cross area A square feet with velocity of V feet per minute, then the flow in one minute is $A\,V$ cubic feet. But in making this statement we assume that the water completely fills the whole of the pipe, so that a cross section perpendicular to the pipe cuts through a water surface of A square feet, and that the velocity of every particle in the stream of water is V feet per minute. The same statement may be made with regard to the flow of water through an orifice, and in making it we make the same assumptions.

Now experiment has shown that these assumptions are by no means accurate. The stream of water flowing through an orifice does not by any means necessarily fill the orifice, nor is its cross section of the same area as the orifice. Under certain conditions the stream may be hollow, and under others it may

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be smaller than the orifice, evidence of which the student will have already discovered if he watched the shape of the stream in the experiment just described. Then the water may form in little whirlpools and eddies at the orifice, in which case the velocity of every particle will be by no means the same, or in the same direction. There is, however, a way in which some idea of the velocity can be obtained. We know that if a body falls freely under the action of gravity through a distance h feet, then its velocity in feet per second is equal to $\sqrt{2gh}$; and one way in which the velocity of water flowing through an orifice is determined is to say that each little particle falls under the action of gravity from the surface of the water to the orifice, where its velocity is therefore $\sqrt{2gh}$ feet per second, when h = depth of the orifice below the surface. But in this statement we take no account of the mass or density of the water, and we should say exactly the same of mercury, which is some $13\frac{1}{2}$ times as heavy as water. We are quite sure that the pressure at any point below the surface of mercury is very much greater than the pressure at a point the same depth below the surface of water, and we know too that flow of water may be due as much to difference of pressure as to difference of head, and that we cannot tell the velocity at any point in a steady stream of water unless we know the pressure at that point. With the stream or jet of water flowing through an orifice we know that the pressure on the outside of the jet is that of the atmosphere, but we do not know the pressure within the jet, and we have no right to say that the velocity is only due to the attraction of gravity, and is equal to $\sqrt{2gh}$ where h is the head.

However, this is a more difficult matter than elementary students can be expected to fully investigate, although we hope they recognize the need of their investigating it some time. It happens also that the points referred to are not so much present with small orifices, and when these are small we are nearer the truth in saying that the average velocity of the stream is $\sqrt{2gh}$ feet per second.

But what about the area of the orifice, or of the stream of water passing through it? It is possible to measure the diameter of the stream or jet by an arrangement of screws

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contained in a frame surrounding the jet, but this is a somewhat more difficult matter than elementary students can deal with. Yet we must not fail to note that if we could actually measure the diameter of jet, and thence find its sectional area, that we could find a useful ratio known as the "*coefficient of contraction.*" This ratio is equal to $\frac{\text{area of stream or jet}}{\text{area of orifice}}$.

Let Q = actual flow of water in cubic feet per second through an orifice whose area is A square feet, and whose coefficient of contraction is K_c, and let V = actual velocity of stream in feet per second. Then as $Q = A K_c V$, we see that V can be determined. Hence if we know the area of the orifice, the coefficient of contraction, and the actual quantity of the flow, we can easily determine the actual average velocity of the stream. It is then quite easy to see how this differs from the velocity as calculated by assuming it to be equal to $\sqrt{2gh}$.

EXPERIMENT II. **To determine the volume of water flowing through a certain orifice, and to find the ratio of this volume to the volume which would flow through, assuming that $Q = A\sqrt{2gh}$ where Q = volume in cubic feet per second, A = area of orifice in square feet, and h = head in feet.**

Method. (1) Measure orifice and find its area. (2) Find the volume of water which flows through the orifice in a given time under *three or five different heads*, exactly as Experiment I. (3) Calculate the volume which would flow through in the same time and under each head, assuming $Q = A\sqrt{2gh}$, and find the ratio of the actual volume to this volume in each case.

Coefficient of Discharge. By means of this experiment we have found a ratio which we may describe as the ratio of the actual flow to the calculated flow with the given orifice, and we observe that this ratio does not vary with the head. If Q^1 = actual flow, Q = calculated flow ($Q = A\sqrt{2gh}$), then $\frac{Q^1}{Q} = K$, which is known as the "*coefficient of discharge.*"

Therefore we see that if we know the area of an orifice, the "head" of the water above it, and the coefficient of discharge,

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or K, we can readily determine the actual flow. For $Q^1 = KQ$, $= KA\sqrt{2gh}$. Hence we see the value of such a coefficient, for although it does not tell us anything about the actual velocity or area of the stream, it does enable us to find out what in practice we most want to know, and that is the actual flow under certain known conditions of orifice and head.

The value of this coefficient for different orifices has been the subject of considerable investigation. For circular orifices this coefficient K is very nearly 0·62.

EXPERIMENT III. **Find the coefficient of discharge with orifices Nos. 3, 4, 5, taking three different heads in each case.**

Method. Proceed as in Experiment II.

EXPERIMENT IV. **To compare the quantity of flow through short pipes—(*a*) parallel bore, (*b*) decreasing bore, (*c*) increasing bore; with the same head, and to find the coefficient of discharge in each case.**

Method. Proceed exactly as in Experiments II. and III. with each orifice, finding the flow under the same three heads in each case. Then compare the results, and calculate the mean coefficient of discharge for each pipe.

Flow of Water through Pipes. The flow of water through pipes is also an extremely important and interesting part of the science of hydraulics, but, like many other interesting questions, it is not altogether easy, and as it requires long and numerous experiments for full investigation, it is not quite within the range of elementary students. But it is possible, with the use of the tanks and orifices already described, and of lengths of pipes and bends of different diameters, to determine some very useful facts about the subject. For example, the effect of the length of a pipe on the flow of water through it under a given head can be determined by using different lengths of ordinary iron, gas, or water piping, one end of which is arranged to screw into a flange, which can be bolted to the tank in the same way as the orifice plates. Also by using pipes of the same length and of different

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diameters, the way in which the flow varies with the diameter can also be found. And again, the effect of bends in a pipe can be determined by screwing on ordinary pipe bends to the end of any of the pipes previously used. Also, a pipe may be suddenly increased and decreased in diameter at some part of its length by inserting a short piece of larger or smaller diameter pipe in the ordinary pipe by means of reducing pieces. The following experiments are therefore suggested :—

EXPERIMENT A. **Determine how the flow of water through a pipe varies with the length of the pipe, the head of water and the diameter of the pipe being constant.**

EXPERIMENT B. **Determine how the flow of water through a pipe varies with the diameter of the pipe, the head of water and the length of the pipe being constant.**

EXPERIMENT C. **Determine how the flow of water through a pipe is influenced by bends in the pipe—(1) when one bend is used turning water in one direction, (2) when two bends are used which turn the water in the same direction, (3) when two bends are used which turn the water in opposite directions.**

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CHAPTER XII.

ENERGY OF FLY WHEELS.

Object of Experiments. The object of the following experiments is to give the student some knowledge of how it is that a body in motion possesses what we call energy, how this energy is measured, and why it is fly wheels are used on engines and other machines. Before commencing the experiments, it is very necessary for the student to rightly understand what is meant by energy, and some other things concerning it.

Energy. Energy is capacity for doing work. A body which can do, or is doing, work is said to possess energy.

Potential Energy. The energy which a body possesses may be simply due to its position; as, for example, a stone on the top of a church tower possesses energy because, if allowed to fall to the earth, it could do work in falling. The energy of a body which depends upon its position only is called *potential energy*, or energy due to position.

Potential energy is measured by the work done in raising the body to the height from which it can fall, and this is, of course, equal to the work which the body can do in falling.

If W = weight of body in *pounds*,
h = fall in *feet*,

Then potential energy or work which weight can do in falling = $W\ h$ *foot-pounds*.

Kinetic Energy. A body in motion can do work in being stopped, and it does this work in virtue of its motion. A familiar example is that of a shot fired from a gun. The

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energy of a body which depends upon its motion is called *kinetic energy*, or energy due to motion.

Kinetic energy is measured by the work which the moving body can do in being brought to rest.

If W = mass of body in *pounds*,
v = velocity of body in *feet per second*,
g = acceleration due to gravity in *feet per second, per second* usually taken as 32·2,

Then kinetic energy or work which body can do in coming to rest $= \frac{W v^2}{2g}$ *foot-pounds*.

Change of Form of Energy. Potential energy can be changed to kinetic energy, and kinetic energy to potential energy. A weight supported at a height above the ground possesses potential energy; if allowed to fall its energy changes to kinetic, so that when half-way down, half its energy is potential and half kinetic, and when it just reaches the ground all its potential energy has disappeared and is changed into kinetic energy, the change having taken place without any loss. Energy may also take other forms, as, for example, when expended in driving an engine, the bearings of the engine become heated, and part of the energy is dissipated as heat; but there has been no loss, for energy cannot be destroyed. This principle is known as that of the *conservation of energy*.

Experiment with Fly Wheel.

Apparatus. (Fig 75.) A cast iron disc fly wheel with a heavy rim is fixed on a steel spindle and mounted between conical bearings in a strong cast iron bracket bolted to the wall, the spindle of the wheel being horizontal. The wheel is 15″ outside diameter, the rim being $2\frac{3}{4}$″ wide and $2\frac{3}{4}$″ thick, the centre disc is $\frac{1}{4}$″ thick, the boss 3″ diameter and 3″ long, and the spindle $1\frac{1}{8}$″ diameter and 12″ long, the whole weighing 96 lbs. The wheel spindle is about 5′ from the floor, and a cord is wound round the spindle so that when weights are tied to the other end they can act on the wheel when falling to the floor, or can be raised by the wheel from the floor. The fixing of the cord on the spindle must be so arranged that it can

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easily free itself as soon as the weight reaches the floor, or can remain fastened so that the cord winds up on the spindle as the wheel rotates and thereby lifts the weights. It is necessary to be able to count the revolutions of the wheel, and this is best done by painting a white line or spot upon the rim, and counting the number of times this passes a small rod fixed just in front of the wheel rim. It is better to arrange for the weight to fall a distance of 8′ or 10′, which can be done by allowing it to pass through the floor. A stop watch is required in certain of the experiments for accurate timing.

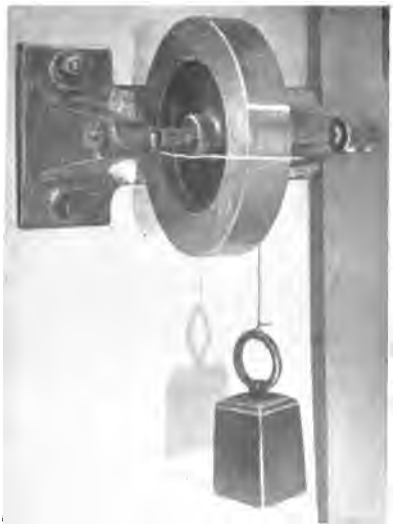

FIG. 75.

Method of Experimenting. Attach a weight to the end of the cord, and starting with the cord well wound up on the spindle, allow the weight to fall to the floor, rotating the wheel as it does so. Now consider the work which the weight has done in falling. We know three things that have happened: (1) work has been done in overcoming the friction of the wheel spindle in its bearings; (2) work has been done in imparting kinetic energy to the wheel; (3) work has been done on the floor by the weight striking it on being brought to rest. We also know that the sum of these three quantities must be equal to the work done by the weight in falling, and therefore that the kinetic energy of the wheel when the falling weight ceases to act upon it is equal to the work due to the fall of the weight *less* the work done in overcoming the friction of the wheel during the fall of the weight, *less* the kinetic energy of the weight at the end of its fall.

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In order to find the friction of the wheel and the kinetic energy of the weight we experiment as follows:—

EXPERIMENT I. **To find the velocity of the falling weight at the end of its fall.**

Method. (1) Hang a weight of 7, 14, 21, or 28 lbs. on the end of the cord. (2) Wind up the cord on the spindle until the weight is at a certain known distance, say 4 feet from the floor, and arrange the fastening of the cord to the spindle so that immediately the weight touches the floor the cord is set free. The required velocity can then be found as follows:

(a) Measure time of fall of weight in *seconds* $= t$.
,, distance ,, ,, *feet* $= h$.

Then velocity of weight at beginning of fall $= o$.
And ,, ,, end ,, $= v$.

$\therefore$ average velocity of weight during fall $= \frac{o+v}{2}$ feet per second.

But space passed through = average velocity × time.

$\therefore h = \frac{o+v}{2} \times t$.

$\therefore v = 2\frac{h}{t}$ *feet per second.*

(b) Measure time of fall of weight in *seconds* $= t$.

Count number of revolutions of wheel during fall of weight $= n$.

Let n^1 = number of revolutions of wheel per second at end of fall.

Then average number of revolutions per second × time of fall in seconds = total revolutions of wheel during fall.

$\therefore \frac{o+n^1}{2} \times t = n$.

$\therefore n^1 = 2\frac{n}{t}$.

Now measure the fall of the weight for one revolution of the wheel. This is best done by revolving the wheel say 10 or 12 times, and measuring the total fall of the weight.

Let h^1 = fall of weight in *feet* for one revolution of the wheel.

Then velocity of weight at end of fall $= h^1 \times n^1 = h^1 \times 2\frac{n}{t}$ *feet per second.*

(c) Measure the time from start to stop of the wheel in seconds $= t^1$.

Count the total number of revolutions of the wheel from start to stop $= N$.

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Let n^1 = number of revolutions of wheel per second at end of fall as before.

Then $\frac{o+n^1}{2} \times t^1 = N$.

$\therefore\ n^1 = 2\frac{N}{t^1}$.

Find the velocity of the weight at end of fall from this result as in Method *b*.

NOTE.—These three methods of finding the velocity of the falling weight at the end of its fall should be tried with three or four different weights on the end of the cord. The following are results obtained with experiments where the fall of the weight was $3\frac{1}{2}$ feet.

	7 lbs. ft. per sec.	14 lbs. ft. per sec.	21 lbs. ft. per sec.	28 lbs. ft. per sec.
Method *a*	0·238	0·476	0·579	0·666
b	0·238	0·475	0·578	0·665
c	0·250	0·480	0·627	0·729
Mean	0·242	0·477	0·595	0·687

EXPERIMENT II. To find the work done in overcoming the friction of the wheel spindle while the weight is falling.

Method (*a*). Hang sufficient weights on the cord so that when given a slight start they will move down uniformly. These weights are the measure of the friction of the wheel.

If weights = *w pounds*, and fall of weights = *h feet*, then work done by weights against friction = $w \times h$ *foot-pounds*.

Method (*b*) Hang on the cord a weight of say 14 lbs., and allow the weight to fall to the floor, revolving the wheel as it does so, as in Experiment I.

Count the total number of revolutions of the wheel from start to stop = N.

Let weight = W *pounds*, and fall of weight = *h feet*.

Then the work done upon the wheel by the falling weight is equal to the work done by the weight in falling (Wh, *foot-pounds*), less the kinetic energy of the falling weight when it reaches the floor $\left(\frac{Wv^2}{2g}\right.$ *foot-pounds*). The method of finding the value of v in this expression in order to calculate this kinetic energy has already been explained in Experiment I.

Therefore work done upon wheel by weight W in falling $= Wh - \frac{Wv^2}{2g}$ *foot-pounds*.

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All this work is expended in overcoming the friction of the wheel spindle from start to stop; for while the weight W acts on the wheel it is overcoming the friction of the spindle, and giving kinetic energy to the wheel; and after the weight has ceased to act this kinetic energy is all expended in overcoming the friction of the wheel spindle.

Therefore average amount of work done in overcoming friction of wheel for one revolution

$$= \frac{W h - \frac{W v^2}{2g}}{N} \text{ foot-pounds.}$$

Hence if we know the number of revolutions of the wheel during the fall of the weight, we can find the work done in overcoming the friction of the wheel spindle for that period.

Results of Actual Experiment of Wheel. Experiment on the wheel described showed that the force required to overcome the friction of the wheel spindle, as in Method *a*, was equal to the weight of 3·531 lbs., which gives the work done against friction during the fall of the weight W through $3\frac{1}{3}$ feet as being $3{\cdot}531 \times 3\frac{1}{3} = 11{\cdot}77$ *foot-pounds*. By Method *b* the work was as follows:—

Falling Weight W.	7	14	21	28 *pounds.*
Work done in overcoming friction per revolution .	1·252	1·251	1·212	1·205 *foot-pounds*
Work done in overcoming friction during fall of weight W . . .	13·146	13·135	12·726	12·652 *foot-pounds*

Now if we compare the results obtained by these two methods, we cannot fail to see that the difference between them is too considerable to justify our taking the mean of the two as being the actual work done in overcoming the friction of the wheel. How can we explain the difference? To answer this let us consider the conditions of the two methods. In Method *a* the weight on the end of the cord was only about $3\frac{1}{2}$ lbs., and the wheel revolved very slowly. In Method *b* the weight on the cord varied from 7 to 28 lbs., and the wheel revolved with considerable velocity. Which of the two variable things, the weight on the cord or the velocity of the wheel, is likely to influence the friction? From our experiments on Friction (Chapter VIII.) we should be inclined to assume that

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the friction would increase proportionately to the increased load on the spindle caused by the weights on the cord, and therefore that the greater these weights the greater the friction. But with the conical lubricated bearings of the wheel spindle the coefficient of friction is sure to be small, and the weight on the cord, even when 28 lbs. is only a small part of the total weight of the wheel causing the friction. On the other hand, we know very little of the influence of the velocity of the rubbing surfaces on the friction produced, except that with lubricated surfaces the friction is probably small at slow speeds, and does not much depend upon the pressure. A glance at the results obtained by Method *b* will tend to confirm this, for there is very little difference in the work done in overcoming the friction for the different weights on the cord. Then again we must feel that the conditions of Method *b* are more nearly like the actual conditions of the experiment than Method *a*; and we feel on the whole that the results of the former are more likely to be accurate than those of the latter.

The student should not, however, be content to accept this conclusion without thoroughly testing it for himself, and satisfying himself by actual experiment on the wheel with which he works as to the conditions of the two methods. The whole experiment is full of the most useful suggestiveness.

EXPERIMENT III. **To find the kinetic energy of the fly-wheel at the time the falling weight ceases to act upon it.**

The actual experiments with the wheel necessary to find this energy have already been performed in Experiments I. and II., and the results obtained from these can therefore be used.

The kinetic energy of the wheel at the instant the falling weight ceases to act upon it must be equal to the work done by the weight in falling, *less* the kinetic energy of the weight at the end of the fall, *less* the work done in overcoming the friction of the wheel spindle. As already stated, we know that:—

Work done by weight W in falling

$= Wh$ *foot-pounds.*

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Kinetic energy of weight W at end of fall

$$= \frac{Wv^2}{2g} \textit{ foot-pounds.}$$

(The value of v is found from Experiment I.)

Work done in overcoming friction of wheel spindle

$$= n\left(\frac{Wh - \frac{Wv^2}{2g}}{N}\right) \textit{ foot-pounds.} \text{ (Exp. II.)}$$

∴ Kinetic energy of wheel at instant weight W ceases to act upon it

$$= Wh - \left\{\frac{Wv^2}{2g} + n\left(\frac{Wh - \frac{Wv^2}{2g}}{N}\right)\right\} \textit{ foot-pounds.}$$

RESULTS OF EXPERIMENTS. *The following are the results of some experiments with the wheel described :—*

Falling Weight.	7	14	21	28 *pounds.*
Mean velocity of falling weight at end of fall.	0·242	0·477	0·595	0·687 *feet per second*
Kinetic energy of wheel at instant when falling weight ceases to act upon it . . .	11·55	33·48	57·16	80·47 *foot-pounds*

Having obtained results of this character with four or five different weights, we can proceed to a useful investigation as follows :—We know that the kinetic energy of a moving mass varies directly with the square of its velocity when its mass is constant. We have not yet discussed what is the velocity of the wheel, but we are quite sure it must be proportional to the velocity of the weight, and, therefore, that its velocity at the instant the falling weight ceases to act upon it is proportional to the velocity of the falling weight at that instant. This velocity was obtained in Experiment I. Now take a piece of squared paper, and set off distances representing the square of the velocity for each of the four or five cases along the base line, and distances representing the kinetic energy of the wheel along the ordinates. Draw the straight line which best passes through the points. If the experiments have been carefully

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performed, the curve should pass through the origin, and the usual equation to the curve will show that the kinetic energy is proportional to the square of the velocity.

Velocity of Wheel, Radius of Gyration. Now let us consider how to express the velocity of the wheel. It is clear that the velocity of the rim is greater than that of the boss, and that, in fact, the velocity of every particle of the wheel depends upon its distance from the axis of the spindle. Evidently then what we want to find is the length of a radius at which we can suppose the whole mass of the wheel to be concentrated, and the velocity of which is therefore equal to the mean velocity of the wheel. This radius is called the *radius of gyration*, and it can be found by calculation for any wheel if its shape and size is known.

If R = radius of gyration in feet and A = angular velocity in circular measure (circular measure for one revolution = 2π), then velocity of wheel = $R\,A$ *feet per second.* ($A = 2\pi n$ where n = revolutions per second) M = mass of wheel in *pounds.* Then kinetic energy of wheel = $M\frac{R^2 A^2}{2g}$ *foot-pounds.*

Moment of Inertia. An expression which is often used for the kinetic energy of a wheel is one which includes the *moment of inertia* of the wheel. The wheel offers a resistance to rotation about its axis due to its possessing what is called *inertia,* and the moment of inertia of any small particle of the wheel is equal to the mass of that particle multiplied by the square of its distance from the axis. The sum of all such terms as this is called the *moment of inertia* of the wheel, and is usually represented by the letter I.

If I = moment of inertia, M = mass of wheel, R = radius of gyration, then $I = MR^2$. Therefore

$$\text{kinetic energy of wheel} = \frac{I A^2}{2g} \text{ foot-pounds.}$$

EXERCISE.

1. From the results of Experiments I., II., and III. find the radius of gyration and moment of inertia of the wheel.

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The M of a Fly-Wheel. A good many people speak of the M of a fly-wheel. Let us consider what this means, and where it is of any use. In an experiment with the wheel described, the kinetic energy stored in the wheel at the instant the falling weight ceased to act upon it was 80·47 *foot-pounds*, and the wheel was rotating at that instant at the rate of 2·2 revolutions per second, or $2{\cdot}2 \times 60 = 132$ revolutions per minute. Now as its kinetic energy varies directly with the square of its velocity, we see that its kinetic energy at one revolution per minute must be equal to $\frac{80{\cdot}47}{132^2}$, or 0·0046 foot-pounds. This is called the M of the wheel, and since it expresses the kinetic energy of the wheel at one revolution per minute, we see that in order to find the kinetic energy at any other speed, we have only to multiply the M of the wheel by the *square* of the number of revolutions per minute.

EXERCISE.

2. Find the M of the fly-wheel from the results of Experiments I., II., and III.

Use of Fly-Wheels. We think the student cannot have worked through the experiments of this chapter without learning a very great deal about energy and about the use of a fly-wheel. He will have seen that a fly-wheel can absorb energy when work is done upon it in causing it to rotate, and that it then does work in virtue of this energy. Now in a steam or gas engine the turning effort at the crank-pin varies during a revolution, owing to the changing pressure of the steam or gas on the piston, and the position of the driving parts. This means that the engine would work in jerks, and the only way of preventing it doing so is to arrange that energy shall be stored up at the best parts of the revolution, and given out at other parts, and this is done by fixing a fly-wheel to the engine shaft. The fly-wheel stores up energy at one part of the revolution, and gives it out in helping to turn the engine at another part. In this way it steadies the engine, and enables it to maintain a more constant speed. The more irregular the turning effort at the crank, the heavier the fly-wheel, as witness

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the very heavy wheels on gas engines, and such machines as punching and shearing machines. The size of the fly-wheel for any given machine can be calculated if the conditions under which it is to work are known; but the problem is too difficult to come within the scope of this book. It is important for the student to remember that the kinetic energy of a fly-wheel can only be altered in amount by a change of speed; it absorbs energy by increasing its speed; it gives out energy by decreasing its speed. The only other way would be to change the mass of the wheel or the position of the mass relative to the axis, and this has been attempted in certain cases by an arrangement of weights which move to and from the centre of the wheel.

EXPERIMENT IV. **Hang weights to the end of the cord, and arrange the attachment of the cord to the spindle so that after falling a certain distance the weight is drawn up by the cord winding round the spindle again. Measure the distances of fall and rise. Find the kinetic energy of the wheel at the instant the weight reaches the lowest point of its fall, and find what becomes of this energy.**

EXERCISE.

3. A fly-wheel weighing 5 tons has a radius of gyration of 10 feet. The wheel is carried on a shaft of 12 inches diameter, and is running at 65 revolutions per minute. How many revolutions will the wheel make before stopping if the coefficient of friction of the shaft in its bearing is 0·065? (Other resistances may be neglected.)

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APPENDIX I.

PARTICULARS OF SPRING BALANCES, SCALE-PANS, AND WEIGHTS.

Spring Balances. The chief requisites in spring balances, especially when used to measure the pull in an inclined or horizontal cord, are that they shall be small and light, and be graduated to fine divisions. Unfortunately it is difficult to combine these in the same balance, as the first two are not easily combined with the third. For example:—an "Improved Spring Balance," reading to 20 lbs. × ¼ lb., is about 7½" long, and weighs 9 ozs.; and a "Patent Spring Balance No. 1," reading to 20 lbs. × 2 ozs., is 10" long, and weighs nearly twice as much. In using spring balances it is well to remember that the same degree of fineness in reading can be obtained by using heavy weights with "coarse marking," as by using light weights with "fine marking." Thus in measuring a pull of 7 lbs. with a balance reading to 2 ozs. means reading to an accuracy of 1 in 56, which is the same degree of accuracy as in measuring a pull of 14 lbs. with a balance reading to 4 ozs.

The ordinary Salter's Balances, as made by G. Salter & Co., of West Bromwich, are as follows:—

Small Brass Sportsman's No. 5 (see Fig. 6). Made in sizes of 2 and 4 lbs. × 1 oz., and 8 lbs. × 2 ozs., and costing from 2*s.* to 3*s.* each. Also larger sizes reading by ¼ lbs. Sizes 8 lbs. × 2 oz., and 10 lbs. × 4 ozs., are 5½" long, and weigh 4 ozs.

Improved Spring Balances (see Fig. 32). In two qualities, Nos. 1 and 2. Nos. 1 are made in sizes of 6, 8, 10, and 12 lbs. × 1 oz., and costing from 4*s.* to 7*s.* each; in sizes of 8, 10, 14, 20, and 24 lbs. × 2 ozs., costing from 2*s.* 6*d.* to 7*s.* each; and in sizes of from 12 lbs. to 60 lbs. × ½ lbs., costing from 1*s.* 10*d.* to 12*s.* 6*d.* each. Also larger sizes reading by ½ lbs. Size 10 lbs. × 2 ozs. is 10½" long, and weighs 1 lb.; 20 lbs. × 4 ozs. is 7½" long, and weighs 9 ozs.; and 40 lbs. × 4 ozs. is 10½" long, and weighs 1½ lbs.

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Patent Pocket Balance (see Fig. 25, upper balances). Made in sizes of 12 to 25 lbs × ¼ lb., and costing from 8*d.* to 3*s.* each. Size 12 lbs. × ¼ lb. is 3½″ long, and weighs 2½ ozs.

Patent Balance No. 1 (strong and sensitive). Made in sizes of 6 lbs. × 1 oz., 6¾″ plate, costing 4*s.*; of 12 lbs. × 1 oz, 12″ plate, costing 7*s.*; and in larger sizes. These are larger and heavier than other patterns.

New Light Circular Balance (see Fig. 27). Made in sizes of 10 and 15 lbs × ½ oz., with dials 6″ to 7¾″ diam., and costing 5*s.* 6*d.* and 8*s.* each. In sizes of 20 and 30 lbs. × 1 oz., 6*s.* and 8*s.* 6*d.* each; and in sizes of 20, 30, 40, and 60 lbs. × 2 ozs., 6″ diameter dials, costing from 6*s.* to 10*s.* each. Size 40 lbs. × 2 ozs. weighs 1⅝ lbs.

Family Balance (see Fig. 13, for measuring "pushes"). Made in sizes of 7 lbs. × ½ oz., 5″ diameter dial, 7*s.* each; and in sizes of 14 lbs. × 1 oz., 5″ dial, 7*s.* each; and 28 lbs. × 2 ozs., 6½″ dial, costing 8*s.* each. Also in larger sizes. If with enamelled dials and black figures, a great improvement, 6*d.* to 1*s.* each extra.

A useful balance of this type for ordinary weighings is the Butter balance, weighing to 2 lbs. × ¼ oz., 4″ diameter dial, costing 7*s.*

Scale-Pans. Strong tin scale-pans, with chains and hooks, are also supplied by Salter's. Sizes from 5″ diameter upwards, increasing by 1″. Prices from 10*d.* each.

Weights (Salter & Co.). Weights are made in two patterns—bar weights, as in Fig. 57, and ring weights, as in Fig. 75. The former are the better for use with scale-pans, and the latter for hanging. Prices per *dozen* as follows: Bar weights—1 lb., 2*s.* 6*d.*; 2 lb., 4*s.*; 4 lbs., 7*s.* 6*d.*; 7 lbs., 13*s.*; 14 lbs., 22*s.*; 28 lbs., 37*s.*; 56 lbs., 70*s.* Ring weights—1 lb, 2*s.* 9*d*; 2 lbs, 4*s.* 6*d.*; 4 lbs., 8*s.*; 7 lbs., 13*s.* 6*d.*; 14 lbs., 24*s.*; 28 lbs., 40*s.*; 56 lbs., 73*s.* Sets of ounce weights, 1 lb. to ¼ oz., 1*s.* 1*d.* per set. These prices are for unstamped weights.

NOTE.—The prices stated above are subject to 10 % discount.

It is desirable to supply enough weights for the whole class when working with friction boards and machines, and the author has usually found that about one dozen complete sets is sufficient for a class of twenty-five to thirty students. Two each of the 1 lb., 2 lb., and 4 lb. weights should be supplied in each set, with one each of the other weights, and a set of ounce weights; except that half a dozen 56 lb. weights are sufficient. The cost of these twelve sets at the prices named is £6 18*s.* 6*d.* Half the weights should be "ring" and half "bar."

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APPENDIX II.

APPARATUS REQUIRED FOR A CLASS OF TWENTY-SIX STUDENTS WORKING IN PAIRS, FOR THE EXPERIMENTS REFERRED TO IN THE MECHANICS SECTION OF THE ELEMENTARY SCIENCE SYLLABUS ISSUED BY THE INCORPORATED ASSOCIATION OF HEADMASTERS.

NOTE.—The numbers refer to the paragraphs in the Physics section of the syllabus.

The work of separate lessons is shown by the paragraphs separated by the lines drawn across the page. Generally speaking, the apparatus for each lesson is sufficient for thirteen pairs of students working at the same time; but when the actual experiment with the apparatus occupies only a short time, this is not always necessary.

The weights required are as specified in Appendix I, and the cost is not included in the separate sections mentioned.

§ 12. **Resolution of Forces.**

Parallelogram of Forces. 7 Imperial drawing boards, 31″ × 23″ (common quality), which may be clamped to vertical posts, or hung on walls by picture frame hooks, each 5*s.*

Two of the boards to be provided with two pulleys (Fig. 3), one with four pulleys (Fig. 18), the remaining boards to be used with two balances and a weight each (Fig. 6).

8 aluminium pulleys with screw centres, at 2*s.* 6*d.*

8 small brass sportsman's balances, 8 lbs × 2 ozs., at 3*s.*

8 4lb. ring weights.

Total cost £3 19*s.*

---

§ 13. **Equilibrium of Three Forces.**

Triangle of Forces. Boards as in § 12, except that two may be arranged with three spring balances (Fig. 7), when the boards may lie on a table.

Extra cost of two balances 6*s.*

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Model of Crane. Balance on jib reading to 14 lbs. by 1 oz.; balance on tie reading to 8 lbs. by 2 ozs.; angle of jib variable. (Fig. 12). Cost, 25*s.*

A Wall Crane (Fig. 13). Balance on jib reading to 28 lbs. by 2 ozs.; balance on tie reading to 20 lbs. by 4 ozs.; angle of jib variable. Cost, 25*s.*

Pulls in Cords. Use the boards without pulleys of § 12 for experiments where (1) the ends of the cord are fixed, and a weight is tied to the centre or some other part of cord, the angle between the two parts of the cord being variable (Fig 6); (2) a vertical cord with weight at free end is pulled by a horizontal cord to any required position (Fig. 9).

N.B.—The boards with pulleys may be used by taking cords over pulleys, and using weights and scale-pans instead of spring balances. For the two spring balances and weights required for each board use those of § 12.

Simple Roof Truss. This can be made by taking two pieces of wood, 30″ to 36″ long, $1\frac{1}{2}'' \times \frac{1}{2}''$, screwing them loosely together at one end, and tying their lower ends together by a cord in which is tied a spring balance which measures the pull in this tie. Cross pieces of wood must be fixed to each end, to prevent the frame toppling over. A weight of 7 or 14 lbs. is hung from the joint. (See Fig. 22.)

	£	*s.*	*d.*
Cost of cranes	2	10	0
Cost of roof truss	0	5	0

Funicular Polygon. (*a*) Use the four boards without pulleys of § 12, with two weights of 7 and 4 lbs., and three balances each.

(*b*) Use the three boards with pulleys of § 12; the end cords passing over the pulleys with weights attached, the magnitude of which represents the pull in the cords, thus dispensing with use of balances for these cords. Use a balance for measuring pull in middle cord. Tie weights of 7, 4, or 2 lbs. to the cords to represent the forces.

(*c*) Cord with three weights of 2, 4, or 7 lbs., and four balances in cord for measuring pulls. Two or three of these can be arranged by tying ends of cord to convenient position on legs of bench or table.

Cost of extra balances.

(*a* and *b*) 5 8 lbs. × 2 ozs., at 3*s.*
(*c*) 12 12 lbs. × 4 ozs., at 8*d.* } £1 3*s.*

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§ 14. Equilibrium of Four or more Forces.

Polygon of Forces. (*a*) Use board with four pulleys of § 12 with five weights.

(*b*) Use models of cranes of § 13 with cord, to represent chain between jib and tie.

(*c*) Use two boards without pulleys of § 12 with three or four balances each, and one weight, the boards being fixed in a vertical plane.

(*d*) Use two boards without pulleys of § 12 with four or five balances, the plane of the board being horizontal.

---

§ 15. Parallel Forces.

(*a*) *Resolution of a Single Force into two Parallel Forces.* Wood beam about 24″ or 30″ long, $1\frac{1}{2} \times \frac{3}{4}$, with a metal screw eye in each end, the upper edge of the beam being divided into inch-lengths. Two of the balances already used may be hung on hooks from the under side of a shelf, or from the end of wall brackets, and the beam be hung from the balances. One or more weights may be suspended by string from any parts of the beam. (See Fig. 27.) Two or three such beams should be arranged. Cost of three beams 5*s.*

If circular balances (10 lbs. × ½ oz.) are used, the cost of each will be 4*s.* 9*d.*

(*b*) *Resultant of a Number of Parallel Forces.* Simple beams of wood, about 24″ or 30″ long, $1\frac{1}{2}'' \times \frac{3}{4}''$, may be suspended by a cord passing through centre of axis of beam to a spring balance, and weights can be tied on the beam in different position. Distance of weights from axis can be measured with metre scales.

	s.	*d.*
Cost of six beams	10	0
Cost of (*a*) and (*b*)	15	0

---

§ 16. Centre of Gravity.

(*a*) For rods use metre scales.

(*b*) Three each—circle (8″ diam.), triangle (sides, 12″, $9\frac{1}{4}''$, $6\frac{3}{4}''$), irregular figure (about $11\frac{1}{2}'' \times 6''$), cut from tin plate, and painted white, with three holes near edge for suspending with string.

(*c*) Two each—pieces of 1″ board: (1) equilateral triangle, $8\frac{1}{2}''$ side; (2) parallelogram, sides, $17'' \times 9''$, acute angles, 45°; (3) right angled triangle sides, 6″, 8″, and 10″; (4) square prism, cone, cylinder, each about 4″ base, and 8″ axis. (These all for placing on inclined boards.)

(*d*) Two inclined boards. A base board, 24″ long, $7'' \times 1''$; to one end is hinged a board, 30″ long, $7'' \times 1''$, on the upper face of which is fixed a sheet of glass, not less than 8″ long. A strong

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upright, having one edge vertical, is fixed to the other end of base board, and from the top end of this projects a small bracket with a ring, through which a cord passes. One end of this cord is tied to a screw eye in the centre of the end of the hinged board, so that by pulling the cord the board can be raised to any desired inclination. The inclination can be measured by placing a metre scale against the vertical edge of upright.

N.B.—Very good results can be obtained by using a simple piece of board, and raising one end on weights (Fig. 50).

Cost of (*b*) 5*s.*
„ (*c*) 10*s.* } £1 5*s.*
„ (*d*) 10*s.*

---

§ 17. **Principle of Moments. Levers.**

(*a*) Wood levers, about 5 ft. long, $1\frac{1}{2}'' \times \frac{3}{4}''$, with metal centres, and fixed on brackets to walls, as in Fig. 29, cost about 16*s.* If with a movable fulcrum, about 2*s.* 6*d.* more.

Cost of three beams (two fixed and one movable fulcrum) £2 14*s.* 6*d.*

N.B.—Very good work can be done with simple stiff laths, or strips of wood, balanced on edge of a triangular block as fulcrum.

(*b*) Bell crank lever, as in Fig. 32. Cost, 17*s.* 6*d.*

Cost of (*a*) and (*b*) £3 12*s.*

---

§ 18. **Simple Machines.**

(*a*) Cord pulley blocks, 2″ brass sheaves, single sheave 3*s.* each, two sheaves 7*s.* 3*d.* each, three sheaves 8*s.* 6*d.* each. Ordinary sash cord (8*d.* per hank) 2*s.* (Fig. 52).

(*b*) Compound wheel and axle, 18*s.* (Fig. 53).

(*c*) Weston's $\frac{1}{4}$-ton Differential pulley block, with 20 ft. of chain, 16*s.* 6*d.* (Fig. 54).

(*d*) Screw jack ($1\frac{1}{2}''$ diam. tripod jack), 12*s.*; board for load, wood pulley, pulley for force cord, 12*s.* (Fig. 56).

Total £4 18*s.*

Friction (not included in Science Syllabus).

	£	s.	d.
Friction board, $30'' \times 7'' \times 1\frac{1}{4}''$, with ends battened, aluminium pulley, and slider with cover, if of pine 9*s.*, if of mahogany, 10*s.*, say three of each (Fig. 48).	2	17	0
Two gin blocks (single sheave pulley blocks), 6″ diam. of pulley and cord (Fig. 46)	0	10	0
Total	£3	7	0

NOTE.—Pulleys on resolution of forces boards can also be used for friction experiments.

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	£	s.	d.
Total	22	0	0
Add for 1 doz. complete sets of weights	6	18	6
Add for 2 doz. metre scales	2	2	0
Add for 2 doz. scale-pans and chains, various sizes	1	5	0
Total	£32	5	6

N.B.—Spring balances, weights, and scale-pans from Salter & Co., of West Bromwich. Machines from Buck & Hickman, Whitechapel, E. Aluminium pulleys from Baird & Tatlock, London. Metre scales from Rabone & Sons, Birmingham.

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APPENDIX III.

ELEMENTARY LABORATORY COURSE IN APPLIED MECHANICS.

The following experimental course has been found to satisfactorily meet the requirements of evening students preparing for the elementary examination of the Science and Art Department in Applied Mechanics. It includes all the experimental work suggested in the syllabus of that subject. The course is arranged for 30 lessons of 1½ or 2 hours each, and the order of working the experiments may be varied as convenient.

Measurement (2 lessons).

Construction and use of vernier. Use of vernier callipers, and micrometer guage.

Resolution of Forces (5 lessons). Chapters I., II., III., pages 1, 29, 38.

Parallelogram, triangle, and polygon of forces. Crane with and without chain. Funicular polygon. Resultant of forces not meeting at a point. Parallel forces.

Moments (1 lesson). Chapter IV., page 52.

Moment of a force. Simple lever. Bell crank lever.

Elasticity and Strength of Wires (4 lessons). Chapter VI., page 65.

Extension of spring with dead and live loads. Modulus of elasticity of indiarubber and metal wire. Complete test of copper and iron wires. Torsion of wires.

Stiffness and Strength of Beams (4 lessons). Chapter VII., page 96.

Variation of deflection with span, breadth and depth. Stiffness and strength with different methods of fixing. Complete test of a beam to breaking.

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Friction (3 lessons). Chapter VIII., page 109.

Friction of simple pulley. Coefficient of friction for wood on wood, metal on metal (dry), metal on metal (lubricated). Angle of friction for wood on wood.

Machines (4 lessons). Chapter IX., page 130.

Velocity ratio and mechanical advantage of one machine. Mechanical advantage of different machines. Friction of different machines. Mechanical efficiency of different machines. Complete test of a machine.

Mechanisms (4 lessons). Chapter X., page 166.

Velocity ratio and slip of belting. Trains of wheels. Cams. Connecting rod and crank motion.

Hydraulics (2 lessons). Chapter XI., page 183.

Flow of water through an orifice under different heads. Coefficient of flow.

Energy of Fly Wheel (1 lesson). Chapter XII., page 192.

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ANSWERS TO EXERCISES

CHAPTER I.

Page 7.

1. 8 from B to A.
2. 12·61.
3. 27·73.
6. 14·17, 12·083, 9·54.
7. 22·18, 39° 27′.
8. 11·6.
9. 8·89, 76° 57′.
10. 12·26, 55°.
11. $OA = 73·18$, $OB = 81·64$.
12. 21·79, 13·23.
13. $OA = 14·74$, $OB = 10·31$.
14. 15·72, forces 14·5 and 6·06.
15. 0·288.

Page 14.

17. Between 5 and 12 = 90°.
    Between 12 and 13 = 157·37°.
    Between 13 and 5 = 112·63°.
19. $CA = 20$, $AD = 17·32$.
20. 25.

Page 18.

21. Inclined string = 24, horizontal string = 20·78.
22. Inclined string = 27·71, horizontal string = 13 86.
23. 1 ton or $\frac{1}{2}$ ton.
24. Pull = 34·64, peg = 40.
25. In 15″ string = 13·2, in 18″ string = 7·2.
26. In 45° string = 8·96, in 30° string = 7·31.
27. In 6′ string = 44·8, in 8′ string = 33·6.
28. 18 lbs.
29. String = 7·07, nail = 10.
30. Cord = 39 6 lbs., tension equal when angle between cord = 120°, tension greater when angle exceeds 120°.
31. 60° = 0·29, 90° = 0·35 tons.

Page 22.

32. Tie = 2000, jib = 3464·2.
33. 23·38.
34. String = 225, rod = 135.

Page 25.

35. $\frac{1}{10}$ ton, 8 lbs.
36. Tension = 3·85, pressure = 9·23.
38. 107·96 lbs.
39. Force vertical, pressure = 0; force at 60° to plane, pressure = 10·392.
40. Tension in thread = 17·32, reaction of plane = 10, pull on Q = 17·32, pressure on A = 34·64.
41. 100 lbs.

Page 28.

42. Pressure on guide = 1178·1 lbs., force along rod and turning effort = 7166·2 lbs.

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CHAPTER II.

Page 33.

1. Resultant = 3.
2. 6·6 lbs. bisecting angle between forces of 7 and 6 lbs.
3. 16·3.

Page 37.

4. Strut = 8, ties = 5·66, beam = 4.
5. Top bars in compression = 5·196, bottom bars in tension = 5·196, end bars in tension = 6.

CHAPTER III.

Page 39.

4. Wall = 29·75, ground = 104·25.
5. Rod = W, hinge = $\frac{1}{2}$ W, string = 0·866 W.
6. 12 lbs.
7. Chain = 43·18, hinge = 43·18.
9. String = 47·7 lbs.; tension decreases as person ascends steps.
10. 54·6.
11. Cord = 3·6, ground = 8·77.

Page 50.

12. $22\frac{1}{2}$, $27\frac{1}{2}$.
13. Resultant = 21 acting at distance of $\frac{6}{7}$ side of triangle from force 4.
14. Resultant acts $\frac{17}{4\frac{1}{2}}$ of length from force 10.
15. From $5\frac{1}{2}$ weight of plank when over trestle to $\frac{1}{2}$ weight of plank when at other end.
16. Vertical supporting force = 4·66.
17. 150 and 0, 25 and 125.
18. $1\frac{67}{120}$, $1\frac{23}{120}$.

CHAPTER IV.

Page 58.

1. (a) opposite signs, (b) − 36 lbs. ins., (c) at E between C and D where $DE = 8\frac{2}{5}''$.
2. $-10l$, $-20l$, 0, where l = side of square.
3. (i.) 900 lbs. ft., (ii.) 112·5 lbs.
4. $AB = +40$, $CB = -28$, $DC = -20$, sum $= -8$.
5. 36·426 anticlockwise.
6. +45 units.
7. $15\sqrt{3}$ lbs. ft.

Page 62.

8. 0 and 380 lbs.
9. P = prop nearest end, Q = other prop; (a) 36·4, 54·6; (b) 78·4, 12·6; (c) − 5·6, 96·6.
10. 225 and − 5, 50 and 170.
11. 2 ozs., 12″.
12. 4″ from 2 m
13. 2·4 lbs.
14. 3″ from weight.
15. 120 lbs.
16. $\frac{3}{4}$ of length of ladder from other man.
17. 0·641 x from 12 lb. wt., x = distance from middle point to this weight.
18. 1·6 lbs.
19. 10 feet.
20. 2·3″ from the 8 lb. wt.
21. at D where $\frac{BD}{DC} = \frac{9}{7}$, 13·89 from D to A.
22. 41·97″.
23. 22·5 lbs. wt.
24. 32·6″.
25. 600 lbs. ft.
26. 54″.

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27. 1190 and 1140 lbs.
28. 7·5 lbs.; fulcrum = 12·5 lbs., vertically.
29. 7·5 and 4·5 ozs.
31. Half-way between middle of rod and weighted end.
32. 26 lbs.
33. 141·36 lbs.

CHAPTER V.

CURVES TO EQUATIONS.

Page 72.

(*a*) $x=7$.
(*b*) $x=-6$.
(*c*) $x=3\frac{6}{7}$.
(*d*) $x=3$, $y=5$.
(*e*) $x=6$, $y=8$.
(*f*) $x=-2$, $y=4$.
(*g*) $x=3$ or 7.
(*h*) $x=6$ or $-\frac{1}{3}\frac{8}{3}$.
(*i*) $x=\frac{1}{3}\frac{3}{3}$ or $-\frac{1}{3}\frac{1}{3}$.

CHAPTER VI.

Page 91.

1. $\frac{1}{1700}$.
2. 39062·5 lbs.
3. 1,440,000 lbs., 0·6″.
4. 7·64 tons per sq. inch, $\frac{1}{15}\frac{1}{36}$, 26,284,032 lbs.
5. 8052 6 lbs., 18,260·8 lbs.
6. 0·746 sq. inches.
7. 26,880,000 lbs.

Page 95.

8. 12°.
9. 720°.
10. 64,800 lbs.

CHAPTER VII.

Page 107.

1. 1406·2 lbs.
2. 9000 lbs. (breaking).
3. 21,600 lbs. (breaking).
4. 4740·74 lbs.

CHAPTER VIII.

Page 119.

1. 10 tons.
2. 626,928 ft. lbs
3. 0·178.
4. 423·36 ft. lbs.
6. 6·3 tons.

CHAPTER IX.

Page 162.

2. 5 cwts.
3. 93·3 lbs.
4. 20,160 ft. lbs., 16·6 %.
5. 6·125 lbs.
6. 71·7 %.
7. 71·6 and 301·6 lbs.
8. 373·3 lbs., 6″, 320 lbs.
9. 4617·9 lbs.
11. 5028·57 lbs.
12. 120 lbs. wt.
13. 6788·57 lbs.
14. 238·857.
15. 24 feet, 4 sheaved pulleys.
17. 300 lbs., 120 ft.
19. (*a*) 33,929·28 lbs, (*b*) 28,274·4 lbs.
20. 113·14.
24. 6034·28 and 2112 lbs.
26. 8, 30·8 %.
27. 1·113 lbs.

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28. 61·539 % wasted, efficiency 38·461 %.
29. 13·36 lbs.
30. 1200 lbs.

CHAPTER X.

Page 176.

2. 3·12″.

Page 179.

3. 22.
4. $\frac{\text{Drivers}}{\text{Followers}} = \frac{3}{4}$.
5. $\frac{1}{4}$″.
6. Reduced $6\frac{8}{25}\frac{19}{25}$ times.
7. $\frac{\text{Drivers}}{\text{Followers}} = \frac{2}{5}$.
8. $\frac{\text{Drivers}}{\text{Followers}} = \frac{4}{11}$.
9. $\frac{\text{Drivers}}{\text{Followers}} = \frac{3}{5}$.

Page 182.

10. $\frac{\text{Drivers}}{\text{Followers}} = 15$.
11. 785$\frac{5}{7}$ ft. per min., 228$\frac{3}{4}$ revs.
12. $123\frac{9}{17}$.
13. $1\frac{100}{3}$ and $1\frac{100}{12}$, or as 4 : 1.

CHAPTER XII.

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3. 354·66 revs.

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[Specimen Page.]

Initia Latina.

I. Show which of the following Verbs are Transitive, and which are Intransitive—

The girl stands. The boys love the mother. The dog runs. The master teaches the boy. The girl sings. The queen praises the boy. Nauta stat. Puer canit. Puer Juliam amat. Julia currit.

II. Point out the Subject, Object, and Predicate in each of the following, writing the proper letters over each word—

The queen loves the boy. The boy fears the dog. The slave loves the girl. Puella servum timet. Servus canem terret. Homo reginam amat.

III. Translate into English—

1. Servus stat. 2. Servus cănem tĭmet. 3. Hŏmo currit. 4. Cănis hŏmĭnem terret. 5. Puella cănem ămat. 6. Aqua currit. 7. Puer puellam dŏcet. 8. Măgister servum dŏcet. 9. Servus nautam vĭdet. 10. Cănis puellam terret. 11. Hŏmo servum vĭdet. 12. Puella cănit. 13. Păter matrem ămat. 14. Māter fīlium dŏcet. 15. Nauta pugnat.

IV. Translate into Latin—

1. The slave runs. 2. The queen sees the slave. 3. The girl sees the sailor. 4. The man stands. 5. The water runs. 6. The boy sings. 7. The girl sees the water. 8. Caesar rules the land.

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est. Helvetii nostrorum impetus diutius sustinere non poterant. alteri in montem se receperunt: alteri ad impedimenta et carros suos se contulerunt. ab hora septima ad vesperum pugnatum est, nec hoc toto proelio aversum hostem videre quisquam potuit.

297. Dum vires annique sinunt, tolerate labores:
jam veniet tacito curva senecta pede.

298. Darius in fuga, cum aquam turbidam et cadaveribus inquinatam bibisset, negavit unquam se bibisse jucundius. nunquam videlicet sitiens biberat.

299. Quid magis est durum saxo, quid mollius unda?
dura tamen molli saxa cavantur aqua.

300. Catilina a Cicerone consule urbe expulsus est, et socii ejus deprehensi in carcere strangulati sunt.

301. Tempori cedere, id est necessitati parere, semper sapientis est habitum.

302. Fame coacta vulpes alta in vinea
uvam appetebat, summis saliens viribus:
quam tangere ut non potuit, discedens ait:
nondum matura est, nolo acerbam sumere.

303. Seneca haec ad amicum scripsit: Ante senectutem curavi, ut bene viverem; in senectute curo, ut bene e vita decedam.

304. Si Alexander, qui tot gentes armis devicit, etiam animi sui cupiditates vicisset, diutius haud dubie et majore cum gloria vixisset.

305. Praeceptores erudiunt pueros, servi dominis serviunt, cives legibus obediunt.

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TITUS.

321. Titus amor ac deliciae generis humani appellatus est. admonentibus domesticis, quia plura polliceretur, quam praestare posset, non oportere, ait, quemquam a sermone principis tristem discedere. atque etiam recordatus quondam super coenam, quod nihil cuiquam toto die praestitisset, memorabilem illam meritoque laudatam vocem edidit: Amici, diem perdidi!

THE LIMITS OF PLAY.

322. Lusus pueris proderunt; quia pueri post lusus plus virium et acriorem animum afferunt ad discendum. modus tamen sit remissionibus; ne aut negatae odium studiorum faciant, aut nimiae otii consuetudinem afferant.

323. AN OLD HALL.

Quin etiam veterum effigies ex ordine avorum
antiqua e cedro, Italusque paterque Sabinus
vitisator, curvam servans sub imagine falcem,
Saturnusque senex Janique bifrontis imago
vestibulo adstabant, aliique ab origine reges,
martiaque ob patriam pugnando vulnera passi;
multaque praeterea sacris in postibus arma,
captivi pendent currus curvaeque secures,
et cristae capitum et portarum ingentia claustra,
spiculaque clipeique ereptaque rostra carinis.

AN "ADMIRABLE CRICHTON."

324. Eleus Hippias, cum Olympiam venisset, gloriatus est, cuncta paene audiente Graecia, nihil esse ulla in arte rerum omnium, quod ipse nesciret; nec solum has artes, quibus liberales doctrinae atque ingenuae continerentur, geometriam, musicam, litterarum cognitionem et poëtarum, atque illa, quae de naturis rerum, quae de hominum moribus, quae de rebuspublicis dicerentur: sed anulum, quem haberet, pallium, quo amictus, soccos, quibus indutus esset, se sua manu confecisse.

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Populus Atheniensis Phocionem patriâ pepulit. Nep.
The Athenian people drove Phocion from his country.

The **Ablative of Origin** is used with Verbs, chiefly Participles, implying descent or origin:

Tantalo prognatus, Pelope natus.
Descended from Tantalus, son of Pelops.

18.

1. The death of Hannibal freed the Romans from fear.
2. No one is free from blame.
3. We are in need of brave soldiers.
4. They stripped the town of defenders.
5. The Helvetii did not abstain from wrong.
6. Caesar calls the soldiers away from the battle.
7. The praetors kept the crowd from the forum.
8. Tarquin, the last king of the Romans, was expelled from the city.
9. The murderers abandoned their attempt.
10. Hippocrates was descended from a Syracusan family.
11. Caesar cut off the enemy from their supplies.
12. He was descended from Hercules.
13. I will relieve you of this load.
14. Love of virtue ought to restrain us from wrong.
15. We hear that he is descended from an ancient family.

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Each Exercise has two parts (A, B).

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8. Leave nothing undone[1] to avenge your brother.

9. It was[2] all through you that I did not defeat the enemy.

10. We shall not prevent them doing that.

III. Indirect Questions.

15. Clauses containing **Indirect Questions** have a verb in the conjunctive, and are joined by interrogative pronouns or conjunctions with the principal verb:

Quæsivit *cur* hæc *fecissem.*
He inquired why I had done this.

Rogaverunt quando futurum esset ut pons conficeretur.
They asked when the bridge would be finished.

Note 1.—The principal verb need not be of an interrogative character:

Moneo *quid faciendum sit.*
I warn you what you ought to do.

Note 2.—The conjunctions *if, whether*, must never be translated by *si, sive*, but by *-ne, num, nonne*:

Dic mihi *num* valeat.
Tell me if he is well.

Note 3.—For a future conjunctive passive the periphrastic forms *futurum sit, fuerit, esset* (followed by *ut* and conjunctive) must be used.

Note 4.—Nescio quis, nescio quomodo (*some one, somehow*) are treated as simple expressions and do not take the conjunctive:

Nescio quis venit.
Some one came.

[1] Prætermitto. [2] Sto.

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4. Caius swore that he would never do anything that was unworthy of a Roman citizen.
5. The river was so rapid that the army could not cross without great danger.
6. The boy asked me whether the old man had lived all his life at Gades.
7. He advised us to be mindful of the shortness of life.
8. He has been made heir to the whole estate.
9. I hope the poor citizens will be spared.
10. You are weak compared to him.

LXVIII.

1. The Senate was nearly all on the side of Hannibal.
2. The dictator swore that if no one followed he would die alone for his country.
3. He ordered the centurion not to kill the prisoners.
4. Who is there that does not love the old generals of Rome?
5. He gave the soldiers two pounds of corn apiece.

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[Specimen Page.]

ĕquĭtātus,	-ūs,	*cavalry.*
pĕdĭtātus,	-ūs,	*infantry.*
mănus,	-ūs,	*band.*
trĭpertĭto,		*in three divisions.*
quam maxĭmus,	-i -ae -i,	*as great as possible.*
hăbeo,	(2),	*hold (levy).*
convĕnio,	-vēni -ventum,	*assemble.*
conscrĭbo,	-psi -ptum,	*enrol.*
compăro,	(1),	*raise.*
cōgo,	coēgi, coactum,	*collect, compel.*

39. [xxxvi.] *War (Service).*

stĭpendium,	-ii,	*pay, service, tribute.*
missio,	-ōnis,	*discharge.*
mĭlĭtia,	-ae,	*warfare, military service.*
sācrāmentum,	-i,	*oath.*
tīro,	-ōnis,	*recruit.*
vĕtĕrānus,	-i,	*veteran.*
immūnĭtas,	-ātis,	*exemption.*
ēmĕrĭti,	-ōrum	*soldiers who have served their time.*
vexillārii,	-ōrum,	*reserve forces.*
in verba jūro,	(1),	*swear (according to a formulary).*
mĕreor,	-ĭtus,	*serve, deserve.*
mĭlĭto,	(1),	*serve (as a soldier).*

40. [xxxvii.] *War (Camp).*

tăbernācŭlum,	-i,	*tent.*
praetōrium,	-ii,	*general's tent.*
porta dĕcŭmāna,	-ae -ae,	*main gate of camp.*
castra hĭberna,	-ōrum,	*winter camp.*
castra aestīva,	-ōrum,	*summer camp.*
castra stătīva,	-ōrum,	*stationary camp.*
ăpertus,	-i -ae -i,	*open, unprotected.*

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2. What is the difference in meaning between the singular and plural of—comitium, littera, ludus, tabula?

3. Translate—what does it matter to me? accused of embezzlement; a house of marble; the day after the battle; after the rising of the sun; do not lie; more than three months; to Naples; lighter than gold; at least; at length.

4. Explain the forms—qui, sultis, viden, fervit.

5. Translate and comment on—(1) Opus est *properato.* (2) *Parcite procedere.* (3) Non recusavit *quominus* poenam *subiret.* (4) Nullum intermisi diem *quin scriberem.*

6. Turn into *oratio recta*—(1) Dixit eum si hoc diceret, errare. (2) Dixit eum si hoc diceret, erraturum esse. (3) Dixit eum si hoc dixisset, erraturum fuisse.

7. Explain the figures in—(1) Pateris libamus et auro. (2) Insaniens sapientia. (3) Superbos Tarquini fasces. (4) Scuta latentia condunt. (5) Dulce loquens Lalage.

8. Give the constructions with—polliceor, impero, refert, vereor, quum, nē. Distinguish between the transitive and intransitive uses of—fugio, consulo, convenio.

9. What do you mean by—cardinal numbers, consecutive clause, co-ordinate sentence, diaeresis, enclitics, labials?

10. What English words are derived from—templum, metior, sidus, dexter, ambio? What were the original names of the months *Iulius* and *Augustus*?

11. Give an example of *coepi* in passive construction.

12. Translate—I know no one to trust. Do not prevent me from going. Do you know how many years Caesar lived? Who has seen the Pyramids without wondering at them? We are permitted to do this.

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Stems in ο pure

	Stem	Masc. φιλιο	Fem. φιλια	Neut. φιλιο
Singular	*N.*	φίλιος	φιλίᾱ	φίλιον
	V.	φίλιε	φιλίᾱ	φίλιον
	A.	φίλιον	φιλίᾱν	φίλιον
	G.	φιλίου	φιλίᾱς	φιλίου
	D.	φιλίῳ	φιλίᾳ	φιλίῳ
Dual	*N.V.A.*	φιλίω	φιλίω	φιλίω
	G.D.	φιλίοιν	φιλίοιν	φιλίοιν
Plural	*N.V.*	φίλιοι	φίλιαι	φίλια
	A.	φιλίους	φιλίας	φίλια
	G.	φιλίων	φιλίων	φιλίων
	D.	φιλίοις	φιλίαις	φιλίοις

Decline also: δίκαιος, *just*; ὅσιος, *holy*.

Contracted Adjectives

	Stem	Masc. χρυσεο	Fem. χρυσεα	Neut. χρυσεο
Singular	*N.*	χρῡσοῦς	χρῡσῆ	χρῡσοῦν
	V.	χρῡσοῦς	χρῡσῆ	χρῡσοῦν
	A.	χρῡσοῦν	χρῡσῆν	χρῡσοῦν
	G.	χρῡσοῦ	χρῡσῆς	χρῡσοῦ
	D.	χρῡσῷ	χρῡσῇ	χρῡσῷ
Dual	*N.V.A.*	χρῡσώ	χρῡσώ	χρῡσώ
	G.D.	χρῡσοῖν	χρῡσοῖν	χρῡσοῖν
Plural	*N.V.*	χρῡσοῖ	χρῡσαῖ	χρῡσᾶ
	A.	χρῡσοῦς	χρῡσᾶς	χρῡσᾶ
	G.	χρῡσῶν	χρῡσῶν	χρῡσῶν
	D.	χρῡσοῖς	χρῡσαῖς	χρῡσοῖς

Decline also: ἁπλοῦς, *simple*; ἀργυροῦς, *of silver*.

Note 1.—Adjectives in *-ους pure* (like ἀργυροῦς) keep the α all through the Feminine Singular.

Note 2.—The methods of contraction should be carefully noted (cf. p. 5).

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53 *ἀγαθῶν*] [R. 113.]

54 *ἀντελάβετο*] lit. 'took by the hand,' *i.e.* 'helped.' [R. 112.]

55 *καθώς...ἡμῶν*] These words form a parenthesis; 'that He might remember mercy (*even as He spake unto our fathers*) toward Abraham and his seed for ever.'

αἰῶνα] Cf. § 121. 46.

48. 1 *ἐγένετο...ἐξῆλθεν*] *ἐγένετο* is a translation of a Hebrew formula of transition; the verb which follows is sometimes connected with *καί*, sometimes, as here, has no connecting particle.

ἐν ταῖς ἡμέραις ἐκείναις] *i.e.* at the time of or shortly after the Annunciation.

ἀπογράφεσθαι] Either passive, 'should be enrolled,' or middle, 'should enrol themselves.' The *ἀπογραφή* was a registration, generally for purposes of taxation. Every Roman subject was liable to a capitation tax.

πᾶσαν τὴν οἰκουμένην] *sc.* *γῆν*, 'all the habitable world,' *i.e.* the Roman Empire.

2 *αὕτη...Κυρηνίου*] 'this was the first enrolment made,' lit. 'this enrolment took place as the first.' Quirinius was Governor of Syria in A.D. 6, ten years after this time. St. Luke has therefore been charged with a grave error in assigning the enrolment to Quirinius. It is probable however that Quirinius was twice governor of Syria, once in B.C. 4, when he began the census, and once in A.D. 6, when he carried it to completion.

τῆς Συρίας] [R. 95.]

3 *ἀπογράφεσθαι*] Infinitive of purpose.

4 *Βηθλεέμ*] 'the house of bread.' Joseph might have got himself registered at Nazareth, but the Jewish practice seems to have been to go to one's native town. The birth of Christ

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5. Translate and comment on—(1) ἀκούσας δὲ αὐτῶν τρυζόντων...τῶν οἰκιῶν ὑμῶν ἐπιπριαμένων. (2) οὐκ ἀνέξομαι ζῶσα. (3) οὔ σοι μὴ μεθέψομαί ποτε. (4) ᾔδει ἄξιος ἂν ὢν θανάτου. (5) γραφὴν ἐδίωκε. (6) ἀμείβειν χρύσεα χαλκείων. (7) πυραμὶς μείζων πατρός. (8) ὁ μάντις τοὺς λόγους ψευδεῖς λέγει.

6. What notion generally precedes the use of πρίν?

Give rules for the construction of final sentences in Greek, with examples.

7. Give the Greek of—mast; sail; anchor; stern; the school of Plato; some people; with impunity; as far as was in their power; may you be happy; skilful in speaking; it being lawful; more honest than rich; fairer than any before; too heavy for a boy; we must obey him; he did it unseen; don't talk.

8. Is there any connection or similarity between the case-endings of Latin and Greek?

9. Translate—

1. He sent for his wife and her son.
2. Do not go away till I come.
3. Surely you do not say so?

LXVIII.

1. Give the Genitive and Gender of—γάλα—ἐλπίς—ἄνθος—πίναξ—κράτος—σάρξ—φέγγος—χρώς—γενειάς—χελιδών.

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il ira	il irait	qu'il aille	qu'il allât	
nous irons	nous irions	que nous allions	que nous allassions	allons
vous irez	vous iriez	que vous alliez	que vous allassiez	allez
ils iront	ils iraient	qu'ils aillent	qu'ils allassent	
je m'en irai	je m'en irais	que je m'en aille	que je m'en allasse	
tu t'en iras	tu t'en irais	que tu t'en ailles	que tu t'en allasses	va-t'en
il s'en ira [irons	il s'en irait	qu'il s'en aille	qu'il s'en allât	
nous nous en	n. n. en irions	que n. n. en allions	que n. n. en allassions	allons-nous-en
vous vous en irez	v. v. en iriez	que v. v. en alliez	que v. v. en allassiez	allez-vous-en
ils s'en iront	ils s'en iraient	qu'ils s'en aillent	qu'ils s'en allassent	
je battrai	je battrais	que je batte	que je battisse	
				bats
				battons
				battez
je boirai	je boirais	que je boive	que je busse	
tu boiras	tu boirais	que tu boives	que tu busses	bois
il boira	il boirait	qu'il boive	qu'il bût	
nous boirons	nous boirions	que nous buvions	que nous bussions	buvons
vous boirez	vous boiriez	que vous buviez	que vous bussiez	buvez
ils boiront	ils boiraient	qu'ils boivent	qu'ils bussent	
je connaîtrai	je connaîtrais	que je connaisse	que je connusse	
tu connaîtras	tu connaîtrais	que tu connaisses	que tu connusses	connais
il connaîtra	il connaîtrait	qu'il connaisse	qu'il connût	
nous connaîtrons	n. connaîtrions	que n. connaissions	que n. connussions	connaissons
vous connaîtrez	vous connaîtriez	que vous connaissiez	que vous connussiez	connaissez
ils connaîtront	ils connaîtraient	qu'ils connaissent	qu'ils connussent	
je courrai	je courrais	que je coure	que je courusse	
				cours
				courons
				courez
je croirai	je croirais	que je croie	que je crusse	
		que tu croies		crois
		qu'il croie		
		que nous croyions		croyons
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LXXIII.

1. Compare—bon, mauvais, petit; and give the adverbs derived from these words. Translate—my best book is here; I am much better.

2. Distinguish—il me rit au nez, il rit de mon nez; excellent, excellant; différant, différent; le cours, la cour; le tour, la tour; vers, vert, le ver; faire grâce, faire une grâce; un écrivain malheureux, un malheureux écrivain.

3. What is the place of the adverb in a French sentence? Translate—I have slept well.

4. Give the masculine of—actrice, hôtesse, institutrice, bergère, jumelle, vache, de laquelle, joyeuse, grasses, sotte, citoyenne; and the plural of—joujou, nez, chacal, sous-officier.

5. What tenses are formed from the present participle? Give examples, and any exceptions you know.

6. Translate—

1. I have passed you the salt.
2. Have you left the door open?
3. I have given your father the book I promised him.
4. Who is there? It is he.
5. I will give it him if you like.

7. Write the infinitive of—mis, sert, envoient, dû, fait, vu, ouvert.

8. Derive—agneau, aigu, ajouter, âme, arriver.

9. Why should the first person plural of *gémir* end in *-issons*, and that of *sentir* in *-ons*? What was the old form of *il aime*?

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